# FEASIBILITY OF USING SATELLITES IN NON-EQUATORIAL, ELLIPTIC, 24-HOUR ORBITS FOR UNINTERRUPTED COMMUNICATIONS

*by*

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**JANUARY 1979**

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A Thesis Submitted
In Partial Fulfilment of the Requirements
for the Degree of

MASTER OF TECHNOLOGY

*by*

VINOD KUMAR JOSHI

*to the*

**DEPARTMENT OF AERONAUTICAL ENGINEERING**

INDIAN INSTITUTE OF TECHNOLOGY KANPUR

**JANUARY 1979**

# CERTIFICATE

This is to certify that the work FEASIBILITY OF USING SATELLITES IN NON-EQUATORIAL, ELLIPTIC, 24-HOUR ORBITS FOR UNINTERRUPTED COMMUNICATIONS has been carried out under my supervision and has not been submitted elsewhere for a degree.

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## ACKNOWLEDGEMENT

The author wishes to record his gratitude to thesis advisor, Dr. Krishna Kumar for suggesting this interesting problem and for his constant encouragement and guidance throughout the course of this work.

The author also wishes to convey his thanks to his friends for their cooperative help during this work. He also wishes to thank Mr. S.S. Pethkar for his excellent typing. Thanks are also due to Mr. Warrayam Singh of Computer Centre.

V. K. JOSHI

## ABSTRACT

It is proposed to explore the feasibility of using 24-hour, elliptic inclined orbits for satellite communications. The study has been initiated with an analysis of the effect of orbital geometry on the various important aspects related to communications satellites, e.g., boosting energy requirements, visibility and changes in peak power and total energy required for communications. The analysis shows that the use of the proposed non-stationery orbit can lead to significant fuel savings. This may however result in some increase in communication power requirements.

The major challenge in the use of the proposed concept appears to be in designing a suitable passive attitude control scheme which can provide for the changing satellite attitude so as to achieve its fixed orientation with respect to the moving line of sight. Fortunately, it has been possible to show that the solar radiation pressure can be effectively employed for achieving this objective to a high degree of pointing accuracy.

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## LIST OF SYMBOLS

| | | |
|---|---|---|
| $a_1$, $a_2$ | : | radii of parking and final circular orbit, respectively |
| $a_3$ | : | $2a_2 - a_1$ |
| $a_n$ | : | $a_2/r_e = 6.628$ |
| e | : | eccentricity of orbit |
| i | : | inclination of the orbital plane with respect to the equatorial |
| $\ell_j$ | : | distance of geometric centre of the set of control plates $S_j - S_j'$ from the satellite mass centre; $j = 1,2$ |
| $n_2$ | : | $a_2/a_1$ |
| r | : | distance between satellite and centre of earth |
| $r_e$ | : | radius of earth |
| $r_n$ | : | $r/r_e$ |
| t | : | time |
| u | : | $\omega + \theta$ |
| x,y,z | : | principal body coordinates with origin at mass centre, s |
| $A_j$ | : | total of the set of control plates $S_j$-$S_j'$ ; $j = 1,2$ |
| $C_j$ | : | Solar parameter $R_p^3\, S\, A_j\, \ell_j(1+\rho_j)/(C'\mu\, I)$; $j = 1,2$ |
| $C'$ | : | velocity of light |

| | | |
|---|---|---|
| $E$ | : | communication energy |
| $E_c$ | : | communication energy required for geosynchronous orbit. |
| $E_e$ | : | communication energy required for elliptic, 24-hour orbit. |
| $I$ | : | $I_{xx} = I_{yy} > I_{zz}$ |
| $I_{xx}$, $I_{yy}$, $I_{zz}$ | : | moments of inertia about x,y,z respectively |
| $K_i$ | : | inertia parameter, $\left[1 - I_{zz}/I\right]$ |
| $P$ | : | communication power |
| $P_{max}$ | : | maximum communication power required in case of elliptic, 24-hour orbit. |
| $P_{ref}$ | : | communication power required for geosynchronous orbit |
| $Q_{\psi}$ | : | generalized force on satellite |
| $R$ | : | distance between mass centre and centre of force |
| $R_p$ | : | perigee distance |
| $S$ | : | solar constant |
| $\alpha$ | : | angle between local vertical and line of sight i.e. line joining satellite and ground station, $\psi - \xi$ |
| $\gamma$ | : | $\phi - \alpha$ |
| $\delta$ | : | longitude of the ground station with respect to vernal equinox |
| $\theta$ | : | satellite position angle, as measured from the perigee |

| | | |
|---|---|---|
| $\mu$ | : | gravitational constant |
| $\mu_j$, $\nu_j$ | : | controller gains; $j = 1,2$ |
| $\xi$ | : | angle between z-axis and line of sight |
| $\rho$ | : | distance between centre of mass and ground station |
| $\rho_j$ | : | reflectivity of the control surfaces $S_j - S_j'$ ; $j = 1,2$ |
| $\phi$ | : | solar position angle ; lattitude of the ground station |
| $\phi_c$ | : | reference solar position angle used for initial setting of the control surfaces |
| $\psi$ | : | librational angle |
| $\omega$ | : | argument of perigee measured from line of nodes |
| $\omega_e$ | : | angular velocity of earth |
| $\Delta\tilde{V}_c$ | : | dimensionless velocity increment required for circular orbit |
| $\Delta\tilde{V}_e$ | : | dimensionless velocity increment required for elliptic, 24-hour orbit. |
| $\Delta\phi$ | : | maximum permissible deviation in $\phi$ before permitting the resetting of the control surfaces |
| $\Omega$ | : | position of line of nodes measured from vernal equinox |
| $(\ )'$ | : | $d(\ )/d\theta$ |

subscript 0 refers to initial conditions unless otherwise specified.

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## 1. INTRODUCTION

"What we are building now is the nervous system of mankind, which will link together the whole human race for better or worse, in a unity which no earlier age could have imagined... " said Arthur C. Clarke who first conceived of communication satellites more than 30 years ago.

What is a communication satellite[1] ? The communication satellite can be looked upon as a space relay moving in orbit, receiving information from a ground station and transmitting it to another ground station. In several respects, space communication is simpler than conventional communication which is severely handicapped due to the problems like tropospheric scatter, ionospheric radio or under water acoustic communications. As a matter of fact, today - 30 years after the prophecy by Clarke - the satellites appear to have already become an important link in global communication network.

The various satellite systems which have been proposed for communications are :

(i) Passive reflector satellites,

(ii) Active satellites in low and medium altitude orbits, and

(iii) Active satellites in a 24 -hour synchronous orbits.

The merits and demerits of the various satellite communication system have been examined by several investigators[2-4]. Of the several concepts proposed for space communication, the one most commonly used

and now almost universally accepted requires placing the satellites into perfect geostationary synchronous orbits. The single over-riding consideration that seems to have weighed heavily in its favour is that satellites that appear absolutely stationary in the sky from the earth allow the use of simpler, less expensive ground stations in communications network. However, there are some other important considerations which appear to have been ignored. This may perhaps be attributed to the undue emphasis being laid on the aspect of operational simplicity. The technological problems associated with the non-stationary satellite alternatives and the likely difficulties of excessive demand on altitude control in ecliptic orbits may have been other possible factors.

The synchronous geostationary satellites although ideally suited to uninterrupted communications are difficult to achieve and maintain particularly over the extended periods of operations. This difficulty arises due to several reasons :

(i) Large orbital deviations are caused due to the various perturbing forces. The frequent orbital corrections are therefore needed leading to rather stringent station-keeping requirements if the full potentialities of the geostationary satellites are to be realized.

(ii) To inject the satellites into the geostationary orbit, large multistage rockets with high boosting capability are needed. The use of a non-equatorial launching site

(unavoidable if the launching has to be done from within India) leads to further increase in expenditure of energy, the additional energy being required for affecting the necessary final transfer maneuver for orienting the satellite orbit to the equatorial plane. This further affects the overall economics of the geostationary satellite adversely.

It may be pointed out that a recent study by Dial and Cooley[5] has established that the net on-station weight capability of satellite can be increased considerably by using a 24-hour, inclined, elliptic orbit. This analysis is of considerable significance as it suggests the possibility of affecting major fuel saving through the use of inclined elliptic but 24-hour orbit for satellite communication.

Here, it is proposed to explore the alternative possibility of using elliptic, non-equatorial, 24-hour satellites for uninterrupted communications. It is felt that this may in general lead to considerable saving in terms of booster power requirements without compromising to any appreciable extent the communication performance characteristics.

It is felt that the apparently simpler and economical proposition if technically feasible would open up a multitude of possibilities which are particularly significant in the light of growing problem related to the overcrowding[6] of the limited equatorial space with satellites and of course the associated political/legal implications[7].

Hence, it is proposed to study some important aspects related to the use of 24-hour inclined, elliptic orbits for satellite communications.

Chapter 2 examines effect of orbital geometry on the requirements of booster capability, communication power and energy, and visibility of the satellites from ground station.

It may, however, be pointed out that one of the most challenging problems in the use of the proposed orbits for communications is likely to be the control of attitude motion due to excitation induced by eccentricity. This problem is further complicated by continual periodic drift of the satellites towards or away from local vertical through the ground station. This brings us to the important problem of control of satellite orientation along the line of sight i.e. the line joining the satellite and the ground station executing periodic oscillations about the local vertical. This phase of investigation aims at exploring the possibility of using solar radiation pressure for achieving this variable attitude control. The problem has been studied in order of increasing complexity.

In Chapter 3, an attempt has been made to synthesize a suitable but simple control law which can enable the proposed solar controller to orient the satellite along the local vertical with a high degree of pointing accuracy.

Chapter 4 deals with the more complicated problem of developing a suitable, simple solar controller model for achieving the variable attitude control of satellite along the moving line of sight.

## 2. EFFECT OF ORBITAL GEOMETRY ON ENERGY AND COMMUNICATIONS REQUIREMENTS.

The various aspects of satellite motion likely to affect the acceptability of its orbit for communication are

(i) boost- energy requirements,

(ii) visibility of satellite from ground station, and

(iii) communication power and energy requirements.

The study has therefore been initiated with an analysis of the effect of orbital geometry on these factors.

### 2.1. Boost-Energy Requirements :

The analysis compares the effect of orbital eccentricity and inclination on boost-energy requirements for the various 24-hour orbits. Since injection of satellite into the final desired orbit is in general preceded by establishing the circular parking orbit first, attention has been focused on computing the total velocity increment required in transfer of satellite from the parking orbit to final desired orbit. Since the objective of analysis is limited to only a preliminary estimate of impulse requirement and their comparison, the 2-impulse, minimum energy i.e. Hohmann transfer trajectory has been assumed for transfer to the perfect geosynchronous orbit. Furthermore, the final impulse at the apogee of the transfer orbit is modified to provide for the change in inclination of the

orbital plane. In the alternative case of transfer to the general 24-hour elliptic orbit, a single impulse is assumed to act at the perigee (Fig. 2.1) of the desired elliptic orbit.

The dimensionless expressions for the total velocity increment required in transfer from the parking orbit to the final circular or elliptic orbits with common 24-hour period are found to be*:

$$\Delta\tilde{V}_c = \sqrt{\mu/a_2}\Big[\sqrt{2n_2 - 2n_2/(1+n_2)} - \sqrt{n_2} + \sqrt{\{1 - \sqrt{2/n_2 - 2/(n_2+n_2^2)}\}^2 + \{2\sin i/2\}^2}\Big] \qquad \ldots (2.1)$$

$$\Delta\tilde{V}_e = \sqrt{\mu/a_2}\Big[\sqrt{2n_2 - 1} - \sqrt{n_2}\Big] \qquad \ldots (2.2)$$

where $n_2 = a_2/a_1$.

Using these expressions, it is possible to compute the percentage saving involved in the use of the proposed elliptic orbits instead of the perfect circular orbit. These results are shown in Table 1.

It is thus apparent from these results that the use of elliptic inclined orbits can enable a significant saving in booster power requirements. The saving increases with increase in eccentricity as well as inclination.

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* See the Appendix

TABLE 1

PERCENTAGE SAVING IN POWER*

| S.No. | e | INCLINATION, i° = Lattitude of the launching station | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 0 | 5 | 10 | 15 | 20 | 25 | 30 |
| 1 | 0.05 | 2.45 | 74.98 | 86.51 | 90.77 | 92.97 | 94.32 | 95.23 |
| 2 | 0.10 | 4.82 | 56.62 | 74.75 | 82.25 | 86.31 | 88.84 | 90.57 |
| 3 | 0.15 | 7.11 | 43.79 | 64.68 | 74.46 | 80.02 | 83.57 | 86.04 |
| 4 | 0.20 | 9.31 | 35.24 | 56.25 | 67.44 | 74.13 | 78.53 | 81.64 |
| 5 | 0.25 | 11.44 | 29.83 | 49.36 | 61.18 | 68.66 | 73.73 | 77.38 |
| 6 | 0.30 | 13.50 | 26.63 | 43.88 | 55.69 | 63.62 | 69.19 | 73.28 |
| 7 | 0.35 | 15.49 | 24.93 | 39.63 | 50.96 | 59.04 | 64.92 | 69.34 |
| 8 | 0.40 | 17.40 | 24.25 | 36.46 | 46.94 | 54.91 | 60.94 | 65.57 |
| 9 | 0.45 | 19.24 | 24.24 | 34.18 | 43.61 | 51.25 | 57.25 | 62.00 |
| 10 | 0.50 | 20.99 | 24.68 | 32.64 | 40.91 | 48.03 | 53.87 | 58.61 |

* Percentage saving in power, $\Delta\tilde{V}/\Delta\tilde{V}_c = (1 - \Delta\tilde{V}_e/\Delta\tilde{V}_c)$ 100

## 2.2. Visibility of Satellites from Ground Station.

An important condition for use of satellites as communication relay without time delay is its visibility from the ground stations being used for the purpose. This imposes constraints on the maximum permissible orbital eccentricities that can be used with satellites in 24-hour orbits. An attempt has, therefore, been made to obtain a theoretical estimate of this upper limit constraint on eccentricity. By analyzing the geometry of motion (Fig. 2.2) it is easy to see that the visibility of satellite from the ground station is assured provided the following inequality constraint is satisfied

$$\cos \beta > r_e/r \qquad \ldots (2.3)$$

where, $\cos \beta = \bar{r} \cdot \bar{r}_e / (|r| \, |r_e|) \qquad \ldots (2.4)$

On evaluating the expressions for cos β in terms of the orbital elements, this inequality takes the form[8]

$$r_n \left[ \{ \cos u \cos(\Omega - \delta) - \sin u \cos i \sin(\Omega - \delta) \} \cos \phi + (\sin u \sin i) \sin \phi \right] > 1 \qquad \ldots (2.5)$$

where $r_n = r/r_e$

$u = \omega + \theta$

$\delta = \delta_o + \omega_e t$

$\delta_o$ = longitude denoting the initial position of the ground station with respect to vernal equinox

$\phi$ = latitude of the ground station

$$\omega_e t = \left[ - e \sin\theta \sqrt{1-e^2}/(1+e\cos\theta) + 2\tan^{-1}\{\sqrt{(1-e)/(1+e)} \tan\theta/2\} \right] \quad \ldots (2.6)$$

This constraint can also be written as

$$r_n \left[ \cos(u - \omega_e t + \Omega - \delta_o) \cos\phi + \{(1-\cos i)\sin(\Omega - \delta_o - \omega_e t)\sin u\} \cos\phi + \sin u \sin i \sin\phi \right] > 1 \quad \ldots (2.7)$$

It is apparent that the visibility of the satellite would be influenced by its initial longitudinal position with respect to that of the ground station. Normally, the relative position of satellite injection into the final orbit would be so chosen as to ensure the best visibility conditions, leading to a considerable simplification in the above relation

$$r_n \left[ \cos(\theta - \omega_e t) \cos\phi - (1-\cos i)\cos\phi - |\sin i \sin\phi| \right] > 1 \quad \ldots (2.8)$$

Using the expression for $\omega_e t$ obtained earlier, it can be shown that

$$\cos(\theta - \omega_e t) \approx \sqrt{1 - 4e^2 \sin^2\theta} \geq \sqrt{1 - 4e^2} \quad \ldots (2.9)$$

Substituting (2.9) into (2.8), the inequality takes the form

$$\sqrt{1 - 4e^2} > 1 - \cos i + |\sin i \tan\phi| + \sec\phi / \{a_n(1-e)\} \quad \ldots (2.10)$$

where

$$a_n = a_2/r_e$$

$$= 6.628$$

The result is of considerable general significance being independent of the several orbital elements, e.g. $\theta$, $\omega$, $\Omega$, $\delta_0$. Furthermore, it provides useful design information giving the maximum permissible eccentricities, consistent with visibility, as a function of orbital inclination with equatorial plane as well as the latitude of the ground station. Results are presented in Table 2.

From these results, it is apparent that the use of large eccentricities is possible for communications even for relatively large inclinations and for large ground station latitudes. For example, it is possible to ensure visibility for eccentricities upto 0.35 even in cases where i and $\phi$ become as large as $30^{\circ}$.

## 2.3. Communication Power Requirements :

Recognizing that the communication power required for transmission from or to satellite is proportional to the square of the distance between the satellite and the ground station, it can be shown that

$$P_{max}/P_{ref} = \left[ (a_3 - r_e)/(a_2 - r_e) \right]^2 \qquad \ldots (2.11)$$

where

$P_{ref}$ = Communication Power required for geosynchronous orbit.

$P_{max}$ = Maximum Communication Power required in case of alternative elliptic, 24-hour orbit.

This expression can be rewritten in a more convenient form as :

TABLE 2

| S.No. | $i^\circ$ \ $\phi^\circ$ | 0 | 5 | 10 | 15 | 20 | 25 | 30 |
|---|---|---|---|---|---|---|---|---|
| | | MAXIMUM PERMISSIBLE ECCENTRICITIES FOR UNINTERRUPTED VISIBILITY | | | | | | |
| 1 | 0 | 0.477 | 0.477 | 0.476 | 0.476 | 0.474 | 0.473 | 0.470 |
| 2 | 5 | 0.477 | 0.475 | 0.474 | 0.472 | 0.459 | 0.466 | 0.461 |
| 3 | 10 | 0.475 | 0.472 | 0.470 | 0.466 | 0.462 | 0.456 | 0.449 |
| 4 | 15 | 0.472 | 0.468 | 0.464 | 0.458 | 0.452 | 0.444 | 0.434 |
| 5 | 20 | 0.468 | 0.462 | 0.456 | 0.448 | 0.439 | 0.428 | 0.414 |
| 6 | 25 | 0.462 | 0.455 | 0.446 | 0.436 | 0.424 | 0.408 | 0.389 |
| 7 | 30 | 0.454 | 0.445 | 0.433 | 0.420 | 0.404 | 0.384 | 0.359 |

$$P_{max}/P_{ref} = [1 + e/(1 - 1/a_n)]^2 \quad \ldots (2.12)$$

or

$$P_{max}/P_{ref} = [1 + 1.176\ e]^2 \quad \ldots (2.12a)$$

The results obtained using this relation are shown in Table 3. It may be noted that the communication power increases with increase in eccentricity. However, this increase in peak power requirement may not be able to offset the gains of fuel saving as it would also be accompanied by the corresponding decrease in the power required at the points of closest approach. A more realistic comparison would, therefore, be provided by estimate of the total communications energy required.

2.4. Communication Energy Requirements :

A comparison of energy requirements for communications for the various eccentricities can be made using the relations

$$E \propto \int_0^{2\pi} P\, d\theta \quad \ldots (2.13)$$

where the power P can be represented by the relation

$$P \propto (r - r_e)^2 \quad \ldots (2.14)$$

Substituting (2.14) into (2.13) and carrying out the necessary manipulation, we get

TABLE 3

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| e | 0 | 0.05 | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | 0.35 | 0.40 | 0.45 | 0.50 |
| $P_{max}/P_{ref}$ | 1.000 | 1.121 | 1.249 | 1.384 | 1.526 | 1.676 | 1.831 | 1.994 | 2.164 | 2.341 | 2.524 |

TABLE 4

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| e | 0 | 0.05 | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | 0.35 | 0.40 | 0.45 | 0.50 |
| $E_e/E_c$ | 0.999 | 0.999 | 0.995 | 0.989 | 0.98[illegible] | 0.969 | 0.955 | 0.939 | 0.919 | 0.896 | 0.870 |

$$E \;\alpha \int_0^{2\pi} (r^2 - 2r\,r_e + r_e^2)\,d\theta \qquad (2.15)$$

On evaluating the integrals appearing in this expression, it is easy to show that for elliptic orbits

$$E_e \;\alpha\; 2\pi \left[a^2 \sqrt{1-e^2} - 2a\,r_e \sqrt{1-e^2} + r_e^2\right] \qquad (2.16)$$

For circular orbits, i.e. when e = 0, this expression reduces to,

$$E_c \;\alpha\; 2\pi \left[a^2 - 2a\,r_e + r_e^2\right] \qquad (2.17)$$

On dividing (2.16) by (2.17) we get

$$E_e/E_c = \sqrt{1-e^2} + (1-\sqrt{1-e^2})/(a_n-1)^2 \qquad (2.18)$$

or

$$E_e/E_c = \sqrt{1-e^2} + 0.032(1-\sqrt{1-e^2})$$

Table 4 shows the effect of orbital eccentricity on increase in communication energy in elliptic orbits. It may be noted that the total energy requirement shows a slight decrease with increase in eccentricity. The gains offered by this marginal decrease in communication energy are however not very significant. The requirement of increased peak power for communication does appear to an additional problem. However if the uninterrupted communications is not essential there is a possibility of combining the satellite communication during the periods when the satellite is relatively closer to the earth with other mission objectives. In such a situation, the main advantages of elliptic orbits namely the higher peak power requirements for communication can be substantially overcome.

# 3. ATTITUDE CONTROL OF SATELLITES IN ELLIPTIC ORBITS USING SOLAR RADIATION PRESSURE

## 3.1 Introduction

For many space applications such as communications, weather-forecasting, survey of earth-resources, it is necessary to maintain the satellites at a fixed orientation with respect to the earth. The use of active attitude control methods although desirable for achieving high degree of pointing accuracy leads to penalty in terms of increased weight, reduced space- availability on board and limited satellite life-time. That perhaps explains why there have been numerous attempts at developing attitude control methods utilizing environmental forces.

The possibility of using Solar Radiation Pressure (SRP) for attitude control has been explored by several investigators[9-14]. Even though effectiveness of this concept has been clearly established, particularly for satellites in high altitude orbits, it appears to have found only limited acceptance by the spacecraft designers. This lack of enthusiasm for the solar attitude controllers may be partly attributed to the availability of the well proven conventional passive and semi-passive alternatives using gravity gradient, satellite spin, magnetic torquing etc. The complexity of the proposed controller models and their operation has perhaps been a major detrimental factor as well. It seems that from the very beginning, there has been undue

emphasis on using the combination of proportional and derivative feed-back control policies for achieving high degree of pointing accuracy. In the process, certain obvious possibilities emphasizing the simplicity of control and operation appear to have been overlooked.

This chapter investigates the feasibility of developing a simple solar attitude controller for satellites operating in elliptic orbits. Attention is focused on evolving a simple control strategy which can counter the adverse effect of eccentricity normally responsible for worsening of the attitude characteristics of the conventional methods.

## 3.2 Formulation of the Problem :

The formulation begins with the analysis of motion of a cylindrical satellite in elliptic orbit, in the ecliptic plane about the earth-center O (Fig. 3.1). The satellite model considered here carries two sets of plates made of light, rigid but highly reflective material (e.g. aluminized mylar membrane). The two sets are assumed to be mounted with their surfaces normal to the orbital plane. Denoted by x,y,z are the principal coordinate axes of the satellite with origin at the mass-centre s. The angle made by the z-axis with the local vertical defines librational angle, $\psi$ . Using Lagrangian formulation, the governing equations of the planar librations can be written as [10,11]

$$(1 + e \cos\theta)\,\psi'' - 2e(1 + \psi')\sin\theta + 3\,K_i \sin\psi\cos\psi = (R^3/\mu I)Q_\psi \qquad \ldots (3.1)$$

where $Q_\psi$ represents the generalized force due to the SRP. For specular reflection assumed and using the principle of virtual work, the expression obtained for the generalized force takes the form

$$Q_\psi = (S/C')\,[(1+\rho_1)\,\ell_1 A_1 \sin(\theta+\psi-\phi)\,|\sin(\theta+\psi-\phi)| - (1+\rho_2)\,\ell_2 A_2 \cos(\theta+\psi-\phi)\,|\cos(\theta+\psi-\phi)|\,] \qquad \ldots (3.2)$$

substituting the above expression for $Q_\psi$ in (3.1) transforms the equation of motion to

$$(1+e\cos\theta)\,\psi'' - 2e\,(1+\psi')\sin\theta + 3K_i \sin\psi\cos\psi = \{(1+e)^3/(1+e\cos\theta)^3\}[C_1 \sin(\theta+\psi-\phi)\,|\sin(\theta+\psi-\phi)| - C_2\cos(\theta+\psi-\phi)\,|\cos(\theta+\psi-\phi)|\,] \qquad \ldots (3.3)$$

where $C_1$, $C_2$ referred to as the solar parameters in the analysis, are given by

$$C_j = \ell_j A_j (1+\rho_j)\, R_p^3\, S/(\mu I C')\,; \quad j = 1,2 \qquad \ldots (3.4)$$

## 3.3. Effect of Solar Radiation Pressure on Attitude Dynamics.

The governing nonlinear, non-autonomous differential Equation (3.3) does not have a closed-form solution. Numerical techniques were therefore used to evaluate the librational performance of the satellites in various practical situations.

Numerical integration was performed using Adams Extrapolation Method[15] in conjunction with a suitable step-size of 2°.

Figure 3.2 shows the librational response in two typical situations with different orbital eccentricities. To bring out the influence of the SRP clearly, the response for $C_1 = C_2 = 0$, i.e., when the solar effect is ignored, has also been included for comparison. It may be observed that the SRP increases the amplitude of librations when $C_1$ is positive. On the other hand when negative values are chosen for this parameter, the SRP seems to influence the system favourably reducing the maximum amplitude of librational motion.

To understand the dynamical behaviour of satellites over a wide range of system parameters, numerous response plots were obtained. The resulting information is condensed in the form of system plots (Fig. 3.3 - 3.5).

Figure 3.3 shows the effect of eccentricity on the librational amplitude in three different situations. It may be observed that in general the amplitude continually increases with increase in eccentricity. Here, solar parameters $C_1$, $C_2$ and the solar position angle $\phi$ appear to be the other significant parameters of the system affecting the librational motion.

Figure 3.4 indicates the effect of the solar parameters $C_1$, $C_2$ on the librational response. It may be pointed out that in

case of circular orbits and for $C_1 = C_2 = 0$, the satellites with their long axes initially oriented along the local vertical do not execute the librational motion unless disturbed. In contrast the response results for any other non-zero combination of the solar parameters exhibit considerable amplitude of librations thus establishing the detrimental influence of the SRP in the circular orbits. However, it is interesting to note that for elliptic orbits, the minimum amplitude does not correspond to the case $C_1 = C_2 = 0$, i.e., when the solar effect is ignored. Furthermore, through a judicious choice of the solar parameters, it is possible to reduce the librational amplitude substantially. The significance of this result cannot be over-emphasized as it suggests an attractive possibility of countering the adverse influence of eccentricity on attitude motion of satellites.

The effect of the varying solar position angle is indicated in Figure 3.5. It may be noted that the solar position has considerable influence on the librational amplitude of satellites, particularly in elliptic orbits. This suggests that a change in $\phi$ significantly affects the choice of the solar parameters for achieving an improvement in the satellite librational performance.

Thus, the numerical results presented here clearly establish the possibility of achieving considerable reduction of the maximum librational amplitude through an appropriate choice of the solar parameters. This beneficial aspect of the SRP appears to have been

either missed or entirely ignored so far. On the other hand, an impression seems to have gained ground that the uncontrolled solar effect is always detrimental to the attitude performance. This may perhaps partly explain why, in the recent developments of the various solar controller models, the emphasis has been essentially on regulating the solar torque through the moving surfaces using complex feedback control systems.

The system plots obtained here can no doubt be used for selecting the appropriate values of the solar parameters in several cases. However, to numerically generate the extensive data to enable the choice of these parameters covering all possible situations is neither feasible nor desirable. So, it was now proposed to treat the problem analytically with a view to develop the suitable criteria facilitating a judicious choice of the solar parameters.

## 3.4 Synthesis of Control Strategy using Approximate Analytical Approach.

It was felt that in view of the complexity introduced by the nonlinear and non-autonomous nature of the problem, it would be essential to make some reasonable assumptions for treating the equations of motion analytically. It is easy to see that for the practical situation of small amplitude librations assumed here, the governing equation can be written as[16]

$$(1+ e\cos\theta)\psi'' - 2e\psi'\sin\theta + 3K_i \sin\psi\cos\psi = 2e\sin\theta +$$
$$\{(1+e)^3/(1+e\cos\theta)^3\}\,[C_1 \sin(\theta-\phi)\,|\sin(\theta-\phi)|$$
$$- C_2\cos(\theta-\phi)\,|\cos(\theta-\phi)|\,] \qquad \ldots (3.5)$$

It may be pointed out that in general the deterioration of the attitude control performance characteristics is mainly due to the periodic excitation represented by $2e \sin\theta$. Attempt was therefore made to explore the possibility of getting rid of this term with the help of the last two terms representing the solar pressure effects. To achieve this objective, the restructuring of the solar terms was considered so that with a suitable choice of $C_1$, $C_2$ the right hand side in the equation could be forced to disappear. Fortunately, Fourier expansion of the terms appearing within the square brackets in Equation (3.5) suggests that they can be effectively described by their first harmonic[16]. On incorporating this approximation, the Equation (3.3) simplifies to

$$(1+e\cos\theta)\psi'' - 2e\psi' \sin\theta + 3K_i \sin\psi\cos\psi = 2e\sin\theta + (8/3\pi)$$
$$\{(1+e)/(1+e\cos\theta)\}^3 [C_1 \sin(\theta-\phi)\,|\sin(\theta-\phi)|$$
$$- C_2\cos(\theta-\phi)\,|\cos(\theta-\phi)|\,] \qquad \ldots (3.6)$$

On separately grouping the terms involving $\sin\theta$ and $\cos\theta$, it was found that the right hand side in the Equation (3.5) vanishes when

$$C_1 = -0.75\pi\,\{e/(1+e)^3\}\,(1+e\cos\theta)^3\cos\phi$$
$$C_2 = +0.75\pi\,\{e/(1+e)^3\}\,(1+e\cos\theta)^3\sin\phi \qquad \ldots (3.7)$$

These results are of considerable significance, as they clearly bring out the effects of the orbital eccentricity as well as the sun-position angle on the desired instantaneous values of the solar parameters. It may be pointed out that by suitably regulating

the movement of the pairs of the solar surfaces $S_1 - S_1'$ and $S_2 - S_2'$ respectively along the z and x-axes (Fig. 3.1), $\ell_1$, $\ell_2$ and hence $C_1$, $C_2$ can be varied as desired. Thus using these relations, it may be possible to control the solar parameters suitably in order to overcome the adverse effect of eccentricity.

A careful examination of the control relations reveals that the changing satellite position in orbit is likely to be of little consequence to the selection of $C_1$, $C_2$, particularly for low values of orbital eccentricities. Hence, these relations can be considerably simplified:

$$C_1 = -0.75\pi \{e/(1+e)^3\} \cos \phi$$

$$C_2 = +0.75\pi \{e/(1+e)^3\} \sin \phi \qquad \ldots (3.8)$$

To examine the validity of the approximate analytical approach, the above expressions developed for $C_1$, $C_2$ were substituted in (3.3) and the resulting equation of librational motion was integrated numerically. Some typical response results thus obtained are shown in Figure 3.6. It may be observed that the control schemes developed analytically are quite effective in limiting the amplitude of librations to much lower values. Even with the highly simplified control of solar parameters represented by (3.8) where $C_1 = C_1(\phi)$, $C_2 = C_2(\phi)$ , the amplitude reduction is found to be considerable. The more stringent control scheme represented by (3.7), where $C_1 = C_1(\theta, \phi)$, $C_2 = C_2(\theta, \phi)$ further improves the controller effectiveness.

To assess the influence of the important system parameters on controller effectiveness, the response results were analysed in details. Figure 3.7 shows the effect of the solar position angle on the system behaviour. As the position of the sun changes continuously in the ecliptic plane, this study would be particularly useful in predicting the long range satellite librational dynamics. The analysis shows that the changing position of the sun does not significantly alter the attitude control characteristics, particularly for low values of eccentricities.

Figure 3.8 exhibits the effect of orbital eccentricity on maximum librational amplitude. It may be observed that both the control strategies are quite effective in reducing the amplitude of motion. For continuously increasing values of eccentricities, the effectiveness of the control schemes progressively declines. The degradation in performance is particularly severe with the simpler control scheme. This limits the applicability of the $\phi$-sensitive control to only slightly eccentric orbits. However, usefulness of the proposed, control scheme sensitive to both $\theta$ as well as $\phi$, extends to much larger range of eccentricities. It is interesting to note that even for eccentricity as large as 0.3, it can limit the amplitude to well within 5°. In view of the recently proposed use of highly elliptic geosynchronous orbits in order to maximize the net on-station weight capability[5], this study may be particularly significant

Attention was now focused on the comparative assessment of the overall attitude performance characteristics associated with the two proposed control laws for the particular case of satellites in near-circular orbits. It was felt that, in this situation, even though the control operation sensitive to both $\theta$ and $\phi$ provides slightly better pointing accuracy, the $\phi$-based action would result in considerably lower demand on the control system. It may be pointed out that the simple, $\phi$-sensitive control involving the single variable is easier to realize than the other one dependent on both $\theta$ and $\phi$. Furthermore, as the period of terms such as $\cos \phi$ and $\sin \phi$ is one year-much higher than the orbital period of the $\theta$-terms involved in the control relations, it is apparent that another major advantage of the simple $\phi$-sensitive control function lies in its much lower frequency. It may be possible to use this fact to advantage by effectively replacing the continuous periodic control action by a limited number of prespecified discrete operations over each cycle. To assess the effectiveness of this approach, the response plots were obtained for the varying degrees of discretization of the proposed $\phi$-sensitive control criteria. The results of this analysis are shown in Figure 3.9. Here, $\phi_c$ represents the solar position angle with respect to which the solar control surfaces are assumed to be set initially and $\Delta\phi$ denotes the maximum permissible deviation in $\phi$ before permitting the resetting of the solar surfaces. It may be observed that the discretization leads to considerable deterioration in the attitude control characteristics. However, for

values of $\Delta\phi$ representing small changes in $\phi$ of say upto $\pm$ 5°, the minor degradation in librational performance is likely to be well within the tolerance limits for many space applications. This result is of considerable practical significance, as it establishes the feasibility of employing the solar surfaces suitably mounted on satellites to achieve substantial reduction in librational amplitude, even without exercising control during the long intervals extending upto as much as a week. The proposed control approach thus suggests a simple method for testing the concept of achieving librational control using the SRP with minimal changes in satellite design.

## 3.5 Effect of Generalizing the Control Law on Satellite Attitude Performance.

The open-loop control policies suggested here although quite effective in limiting the librational amplitude to low values, does not provide for damping the excited librations. It is, therefore, proposed to combine them with the proportional and derivative control. The resulting control law can be written as follows:

$$C_j = C_{j1} + C_{j2} = C_j(\theta, \phi, \psi, \psi') \qquad \ldots (3.9)$$

where

$$C_{11} = C_1(\phi) \quad \text{or} \quad C_1(\theta, \phi)$$

$$C_{21} = C_2(\phi) \quad \text{or} \quad C_2(\theta, \phi)$$

$$C_{12} = C_1(\psi, \psi') = \begin{cases} -(\mu_1\psi' + \nu_1\psi); & 2k\pi < \theta + \psi - \phi \leq (2k+1)\pi \\ +(\mu_1\psi' + \nu_1\psi); & (2k+1)\pi < \theta + \psi - \phi \leq (2k+2)\pi \end{cases}$$

$$C_{22} = C_2(\psi,\psi') = \begin{cases} +\ (\mu_2\psi' + \nu_2\psi)\ ; & (2k-\tfrac{1}{2})\pi < \theta+\psi-\phi \le (2k+\tfrac{1}{2})\pi \\ -\ (\mu_2\psi' + \nu_2\psi)\ ; & (2k+\tfrac{1}{2})\pi < \theta+\psi-\phi \le (2k+\tfrac{3}{2})\pi \end{cases} \quad \ldots (3.10)$$

with $|C_{j2}| \le C_{j\max}$ , k being an integer.

The various results are obtained by solving Equation (3.9), using the earlier numerical techniques in conjuction with a suitable step size of 0.5°.

Figure 3.10 shows the librational response of satellite in some typical situations. It may be noted that using the control relation $C_j = C_j(\phi,\psi,\psi')$ obtained by the combination of (3.8) and (3.10), leads to significant reduction in the steady state librational amplitude; while (3.9) reduces the librational amplitude even further.

To understand the behaviour of system and control parameters on controller effectiveness, numerous response plots were obtained. The resulting information is condensed in the form of system plots (Fig. 3.11 - 3.14).

Figure 3.11 shows the effect of mass distribution on the steady state librational amplitude in several situations. It may be observed that the inertia parameter, $K_i$ has considerable influence on the amplitude for $\mu_j = \nu_j = 0$, i.e., when only open-loop control $C_j = C_j(\phi)$ or $C_j = C_j(\theta,\phi)$ is employed. For the feedback control represented by nonzero values of $\mu_j$ and $\nu_j$, however, the change in $K_i$

appears to have no effect on the system plots. Furthermore introducing the additional terms to offset the effect of excitation induced by eccentricity reduces the librational amplitude considerably. In general, the amplitude of oscillations is found to be lowest when the more stringent control strategy represented by $C_j = C_j(\theta,\phi,\psi,\psi')$ is employed.

The influence of orbital eccentricity on the amplitude of motion is shown in Fig. 3.12. It may be noted that in general, increase in the eccentricity adversely affects the librational performance. However, the suggested modifications in the control strategy substantially improve the controller effectiveness and limit the amplitude to much lower values even in case of relatively large eccentrities.

Figure 3.13 shows the effect of control gains $\mu_j$ and $\nu_j$ on librational amplitude. It may be observed that a judicious choice of the gains can minimize the attitude variations considerably. Even for the best choice of controller gains, the suggested control modifications appear to bring about a considerable reduction in the amplitude of librational motion. It is apparent that even for eccentricities as large as 0.1, the proposed controller can limit the amplitude approximately to within 0.1°.

Figure 3.14 shows the effect of $C_{jmax}$ and the initial impulsive disturbance $\psi'_o$ . In general an increase in $C_{jmax}$ affects the attitude performance favourably to a point beyond which the

dependence becomes rather weak. It may be observed that with the introduction of the suggested modification in the control law, it is possible to use much smaller values of $C_{jmax}$.

The proposed controller presents an interesting possibility of controlling satellite orientation in elliptic orbits. The realization of the control is easy and the size of the plates required quite modest. A preliminary estimate using the data of INTELSAT IV series of satellites shows that the plate size of 0.4 $m^2$ with permissible movement of 20 cm. is adequate for orbital eccentricities of 0.1. For larger eccentricities, this requirement increases in nearly the same proportion. The semi-passive character of the system promises increased life-time with overall reduction in the cost.

## 3.6. Concluding Remarks.

The important features of the analysis and the conclusion based on them may be summarized as follows:-

(i) Contrary to as is generally believed, the SRP need not be always detrimental to the satellite attitude response. In fact, there are several situations, where for a proper choice of the solar parameters, the radiation pressure can substantially reduce the maximum librational amplitude of satellites in elliptic orbits.

(ii) The system plots, developed numerically, summarize the satellite librational response characteristics over a wide range of parameters. Through these plots, it is possible to select the

appropriate values of the solar parameters minimizing the librational amplitude in several cases.

(iii) The approximate analytical approach adopted here seeks to make use of the solar terms in the equation of librational motion to eliminate the major excitation effect of eccentricity normally responsible for the worsening of the attitude control characteristics of the conventional methods. The two control strategies thus evolved are found to be quite effective in restraining the satellite oscillations.

(iv) The proposed control schemes are much simpler as they are dependent only on $\theta$ and $\psi$, which in turn can be expressed as a function of time alone. This makes them particularly suited for preprogramming of the control function unlike the earlier complex feedback control approaches sensitive to the librational disturbances and requiring on-board sensing and computation of the error signal for actuating the controls.

(v) Using the relatively more stringent control law governed by the instantaneous values of the position of the sun ($\phi$) and satellite ($\theta$) leads to drastic reduction in maximum librational amplitude. It considerably extends the effective range of possible eccentricities for stable satellite operation. It is interesting to find that even for eccentricity as large as 0.3, the proposed controller can limit the maximum amplitude to well within 5°. This suggests the possibility of using it as

an auxiliary control device for improving the overall librational performance characteristics of satellites in highly elliptic orbits. The proposed control mechanism may thus have far reaching implications. In view of its ability to overcome the adverse effect of eccentricity to a large extent; it may lead to greater acceptance of highly elliptic orbits, particularly for multi-space missions.

(vi) The simpler $\phi$-sensitive control strategy is quite effective for satellites in slightly eccentric orbits. Furthermore, it is possible to effectively replace this continuous periodic control action by a limited number of prespecified discrete operations over each cycle. The analysis establishes the feasibility of employing the suitably mounted solar surfaces on satellites to improve the librational performance even without exercising control for long intervals. The proposed control approach thus suggests a simple method for testing the concept of librational control using the SRP with minimal changes in satellite design.

(vii) Combining the suggested open-loop control with proportional and derivative feedback control appears to bring about a considerable reduction in the amplitude of librational motion. For example, the use of the modified control law enables the solar controller to limit the librational amplitude to within approximately $0.1^\circ$, even for eccentricities as large as 0.1.

## 4. ATTITUDE CONTROL OF SATELLITES IN NON-STATIONARY 24-HOUR ORBITS

### 4.1 Introduction

The advantages of having communications satellites in a perfect geosynchronous orbit are well understood. These orbits not only enable the use of simple ground stations but also facilitate the use of relatively simpler subsystems for station-keeping, attitude control and communications. It is, therefore, natural to conclude that any major deviations from the well established practice of using perfect synchronization between the satellite and the ground station is going to pose a major challenge on several fronts and a major effort is called for to meet these challenges successfully.

In particular, the problem of using elliptic, inclined, 24-hour orbits for communications is significantly complicated by continual periodic drift of satellites from their synchronous position directly over the ground station. The non-stationary nature of the satellite causing periodic oscillation of the line of sight about the local vertical leads to a continual periodic change in its orientation as viewed from the ground station.

This problem can perhaps be easily handled through the active attitude control method. However, on-board energy requirements involved here are likely to offset the advantages of economy in booster capacity needed for placing the payload in the desired orbit. It seems, however, possible to overcome this difficulty through the

passive control schemes using solar radiation pressure. It is, therefore, proposed to explore the possibility of evolving a near-passive solar attitude controller which can vary the satellite orientation continuously so as to compensate for the steady or periodic satellite drift relative to the ground station.

## 4.2 Formulation of the Problem :

Geometry of the satellites assumed to be moving in elliptic equatorial orbit is shown in Fig. 3.1. The angle, $\xi$ made by the z-axis with the line of sight defines satellite orientation as apparent from the ground station. Recognizing the relation between this apparent angle, $\xi$ and librational angle, $\psi$ as

$$\psi = \xi + \alpha \qquad \ldots (4.1)$$

where

$$\alpha = \tan^{-1}\{\sin(\theta - \omega_e t)/(r_n - \cos(\theta - \omega_e t))\} \qquad \ldots (4.2)$$

Using (4.1), the Equation (3.3) can be written as

$$(1+e \cos )\xi'' - 2e(1+\xi') \sin + 3K_i \sin\xi \cos \xi$$
$$= \{(1+e)^3/(1+e \cos\theta)^3\}\left[C_1 \sin(\theta+\xi+\alpha-\phi)|\sin(\theta+\xi+\alpha-\phi)|\right.$$
$$\left. - C_2 \cos(\theta+\xi+\alpha-\phi)\,|\cos(\theta+\xi+\alpha-\phi)|\right] - (1 + e \cos\theta)\alpha''$$
$$+ 2e \sin\theta\, \alpha' + 3K_i \sin\xi \cos\xi - 3K_i \cos\psi \sin\psi \ldots \qquad (4.3)$$

## 4.3 Effect of SRP on Attitude Dynamics :

To understand the dynamical behaviour of satellites over a

wide range of system parameters, various response plots were obtained numerically. The resulting information is condensed in the form of system plots (Fig. 4.1 - 4.3).

Figure 4.1 shows the effect of eccentricity on the librational amplitude in three different situations. It may be noted that the maximum librational amplitude increases with increasing value of eccentricity. The effect of solar parameters $C_1$ and $C_2$ on the librational motion is shown in Fig. 4.2. As before, it may be pointed out that the minimum amplitude does not correspond to the case when $C_1 = C_2 = 0$. Furthermore, through a judicious choice of solar parameters, the librational amplitude can be reduced considerably.

Figure 4.3 shows the effect of variation of solar position angle, $\phi$. It may be noted that $\phi$ has considerable influence on librational motion. This shows that a change in $\phi$ significantly affects the choice of the solar parameters to achieve improvement in the satellite librational performance.

In view of the obvious limitations of numerical approach leading to a judicious choice of the various parameters, attempt was now made to use an approximate analytical approach for predicting the suitable choice of control parameters.

## 4.4 Analytical Approach :

Due to complexity and nonlinearity of the nature of the problem several simplifying assumptions have been made to treat the

problem analytically. For small amplitude motion, it is possible to approximate (4.2) by the relation

$$\alpha \approx \sin(\theta - \omega_e t)\ (1 + e \cos\theta)/a_1 \qquad \ldots (4.4)$$

where

$$a_1 = a_n(1 - e^2) - 1$$

Linearizing (4.3) in conjuction with (4.4), equation of motion can be written as

$$(1+e \cos\theta)\xi'' - 2e \sin\theta\, \xi' + 3K_i \sin\xi \cos\xi = 2e(1 - 3K_i/a_1) \sin\theta$$
$$+ \{ (1+e)/(1+e \cos\theta) \}^3 \left[ C_1 \sin(\theta - (\phi - \alpha)) \right.$$
$$\left. |\sin(\theta - (\phi - \alpha))| - C_2 \cos(\theta - (\phi - \alpha))\ |\cos(\theta - (\phi - \alpha))| \right] \qquad \ldots (4.5)$$

Following the steps as outlined in Chapter 3, it can be shown that the effect of excitation due to eccentricity and gravity gradient moment can be counteracted by controlling the solar parameters according to the relations

$$C_1 = -0.75\pi\, e\ (1-3K_i/a_1) \cos\gamma\ (1+e \cos\theta)^3/(1+e)^3$$
$$C_2 = +0.75\, \pi e\ (1-3K_i/a_1) \sin\gamma\ (1+e \cos\theta)^3/(1+e)^3 \qquad \ldots (4.6)$$

where $\gamma = \phi - \alpha$ and $\alpha \approx (\theta - \omega_e t)/a_1$

These results are of considerable significance, as they clearly bring out the effects of the orbital eccentricity inertia parameter and the sun position angle on the desired instantaneous values

of the solar parameters. It may be noted that by suitably moving the solar plates, the solar parameters $C_1$, $C_2$ can be varied in order to overcome the adverse effect of eccentricity.

A careful examination of the control relations reveals that the changing satellite position in orbit is likely to be of little consequence to the selection of $C_1$, $C_2$ particularly for low values of orbital eccentricities. Hence, these relations can be considerably simplified:

$$C_1 = -0.75\pi\, e(1- 3K_i/a_1)\, \cos\phi/(1+e)^3$$
$$C_2 = +0.75\,\pi\, e(1-3K_i/a_1)\, \sin\phi/(1+e)^3 \qquad \ldots (4.7)$$

To examine the validity of the approximate analysis, the above expressions for $C_1$, $C_2$ were substituted in (4.5) and the resulting equation was solved numerically. The results are shown in Fig. 4.4. It may be noted that the control schemes developed are indeed quite effective. However, the approximate control of solar parameters according to $C_j = C_j(\phi,K_i)$ represented by (4.7) has almost the same effect as that due to $C_j = C_j(\theta,\phi,K_i)$ denoted by (4.6).

To understand the dynamic behaviour of system parameters on the controller effectiveness, the response results were analyzed. Figure 4.5 shows the effect of the solar position angle on the maximum librational amplitude. This study may be useful in predicting the long range satellite dynamical behaviour. The analysis shows that there is not much change in the librational amplitude especially for low eccentricities.

Figure 4.6 shows the effect of eccentricity on the maximum librational amplitude. It may be observed that both control strategies are equally effective in reducing the amplitude of motion for the low values of eccentricities. As the value of eccentricity increases, the effectiveness of control schemes decreases. However, it is worthwhile to note that the amplitude can be limited well within $5^\circ$ even when $e = 0.10$.

## 4.5. Effect of Generalizing the Control Law on Satellite Attitude Performance

As the open loop control policies suggested above are not fully effective for higher values of eccentricities, it is proposed to combine them with the proportional and derivative control law. The resulting control law can be written as :

$$C_j = C_{j1} + C_{j2} = C_j(\theta, \phi, K_i, \psi, \psi') \qquad \ldots (4.8)$$

where

$$C_{11} = C_1(\phi, K_i) \quad \text{or} \quad C_1(\theta, \phi, K_i)$$

$$C_{21} = C_2(\phi, K_i) \quad \text{or} \quad C_2(\theta, \phi, K_i)$$

$C_{12} = C_1(\psi, \psi')$ and $C_{22} = C_2(\psi, \psi')$ are given by (3.10)

Since the modified governing equation of motion does not have closed-form solution, numerical techniques were applied to study the system behaviour. Here, a stepsize of 0.5° was found to be suitable for numerical integration.

To examine the suitability of control given by (4.8), some typical response results were obtained (Fig. 4.7). It may be observed that the solar control given by $C_j = C_j(\psi, \psi')$ is quite effective in limiting the librational amplitude. The more stringent control schemes represented by $C_j = C_j(\phi, K_i, \psi, \psi')$ combination of (4.7) and (3.10) and $C_j = C_j(\theta, \phi, K_i, \psi, \psi')$ combination of (4.6) and (3.10) further improves the controller effectiveness.

To assess the effect of system parameters on the steady state librational amplitude, numerous response results were obtained which are presented here in a rather condensed form (Fig. 4.8). It may be observed that with the proportional and derivative control the steady state librational amplitude is reduced considerably. The amplitude can be reduced further using the modified control laws $C_j = C_j(\phi, K_i, \psi, \psi')$ and $C_j = C_j(\theta, \phi, K_i, \psi, \psi')$. Furthermore, the results suggest that the simpler control strategy denoted by $C_j = C_j(\phi, K_i, \psi, \psi')$ is quite adequate and should hence be preferred.

## 4.6. Concluding Remarks:

The important features of the analysis and the conclusion based on them may be summarized as follows:

(i) The proper choice of solar parameters can reduce the maximum librational amplitude substantially.

(ii) The system plots, developed numerically, summarize the satellite librational response characteristics over a wide range of parameters. Through these plots, it is possible to

select the appropriate values of the solar parameters minimizing the librational amplitude in several cases.

(iii) The two control strategies $C_j = C_j(\phi, K_i)$ and $C_j = C_j(\theta, \phi, K_i)$ are found to be quite suitable in overcoming the major excitation effect of eccentricity and inertia parameter.

(iv) The simpler control strategy represented by $C_j = C_j(\phi, K_i)$ can be employed quite effectively for limiting the librational amplitude without appreciably compromising with pointing accuracy. The resulting solar controller can also be used as an auxiliary control device for improving the overall librational performance characteristics of satellites in elliptic orbits if higher degree of pointing accuracy is required.

(v) The addition of derivative and proportional control makes the controller more effective. This generalized control limits the steady state librational amplitude to well within 5° for eccentricities even as large as 0.20.

(vi) The generalised control policy indicated by $C_j = C_j(\phi, K_i, \psi, \psi')$ appears to be the best as it generally leads to minimum limit cycle amplitude of librations.

## 5. CLOSING REMARKS

### 5.1 Summary of the Conclusions

As already stated the study aimed at examining the various aspects of motion related to the use of 24-hour, non-stationary orbits for satellite communication. The important conclusions based on the analysis can be summarized as follows:

(i) The use of the proposed non-synchronous orbits can enable substantial saving in the booster capability requirements.

(ii) Even for relatively large eccentricities, and with moderate orbital inclinations and ground station latitudes, the uninterrupted visibility is possible.

(iii) The requirements of peak power and total energy for communications show a continuous increase with increasing orbital eccentricity. However, this may not pose a major limitation, particularly with proper scheduling of communication periods and when this main objective satellite communication objective is combined with various other important missions, e.g., remote sensing for survey of earth resources and better weather-forecasting scientific space research etc.

(iv) To provide for the changing satellite attitude so as to achieve its fixed orientation with respect to the moving line of sight appeared to be a major challenge involved

involved in the use of the proposed nonsynchronous orbits for communications. The investigation showed that this challenge can be successfully met by employing the passive attitude control method using the solar radiation pressure.

## 5.2. Recommendations for Future Work

There are several aspects related to the use of 24-hour non-synchronous satellites for communications which have not been considered in the present investigation. It would be worthwhile to carefully examine these aspects which may be summarized as follows:

(i) The ground station may require using more sophisticated system. It would, therefore, be essential to examine the effect of the resulting increase in the cost of ground component on overall economics.

(ii) A transmission power control will become necessary to counteract the variation of the range caused by orbital ellipticity.

(iii) Several communication problem related to frequency coordination, intermodulation are likely to be adversely affected. Besides, it would be desirable to extend the analysis to study the attitude control characteristics of the proposed solar model as affected by the orbital inclination, roll and yaw motion.

# APPENDIX

A-1 Calculation of total velocity increment in transfer of satellites from parking orbit to 24-hour coplanar elliptic orbit.

The geometry of satellite motion and its transfer from circular parking orbit to the final 24-hour elliptic orbit is shown in Fig. 2.1. It is easy to show that the circular velocity at P is given by

$$V_{P_c} = \sqrt{\mu / a_1} \tag{A.1}$$

The corresponding velocity at P along Hohmann transfer trajectory is given by

$$V_{P_t} = \sqrt{\mu \left[ 2/a_1 - 2/(a_1 + a_3) \right]} \tag{A.2}$$

The velocity increment required at p for injecting the satellite from parking orbit into the 24-hour elliptic orbit is therefore given by

$$\Delta V_e = \sqrt{\mu / a_2} \left[ \sqrt{2a_2/a_1 - 2a_2/(a_1+a_3)} - \sqrt{a_2/a_1} \right] \tag{A.3}$$

On non-dimensionalising the velocity increment, we get

$$\Delta \tilde{V}_e = \sqrt{2n_2 - 1} - \sqrt{n_2} \tag{A.4}$$

where

$$\Delta \tilde{V}_e = \Delta V_e / \sqrt{\mu / a_2}$$

A.2 Calculation of total velocity increment in transfer of satellites from parking orbit at an inclination with respect to the equatorial plane to geosynchronous orbit.

As shown earliest, velocity increment required at P to change the satellite orbit to Hohmann transfer trajectory can be written as

$$\Delta V_P = \sqrt{\mu / a_2} \left[ \sqrt{2a_2/a_1 - 2a_2/(a_1+a_2)} - \sqrt{a_2/a_1} \right] \quad (A.5)$$

Similarly, the velocity increment required at A in order to inject the satellite into geosynchronous orbit can be obtained

$$\Delta V_A = \sqrt{\mu / a_2} \left[ \sqrt{\{1-(a_1/a_2)\sqrt{2a_2/a_1 - 2a_2/(a_1+a_2)}\}^2 + \{2\sin i/2\}^2} \right. \quad (A.6)$$

where i is inclination of parking orbit, combine (A.5) and (A.6) results in the following non-dimensional expression for the total velocity increment

$$\Delta V_c = \Delta V_P + \Delta V_A$$

$$\Delta \tilde{V}_c = \sqrt{2n_2 - 2n_2/(1+n_2)} - \sqrt{n_2} + \sqrt{\{1-(1/n_2)\sqrt{2n_2 - 2n_2/(1+n_2)}\}^2 + \{2\sin i/2\}^2} \quad (A.7)$$

where

$$\Delta \tilde{V}_c = \Delta V_c / \sqrt{\mu / a_2}$$

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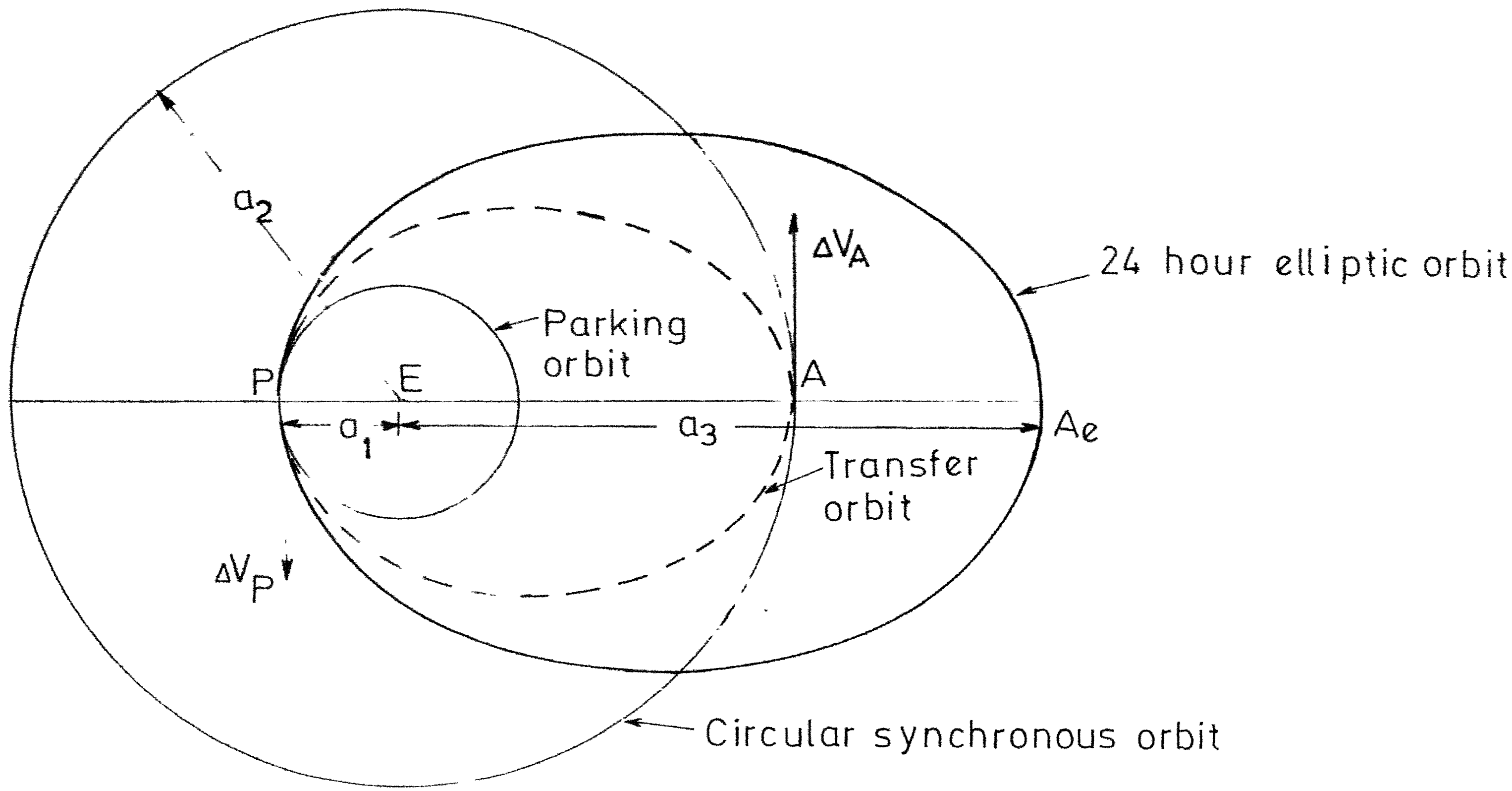


Fig. 2·1 Geometry of 24-hour circular and elliptic orbits

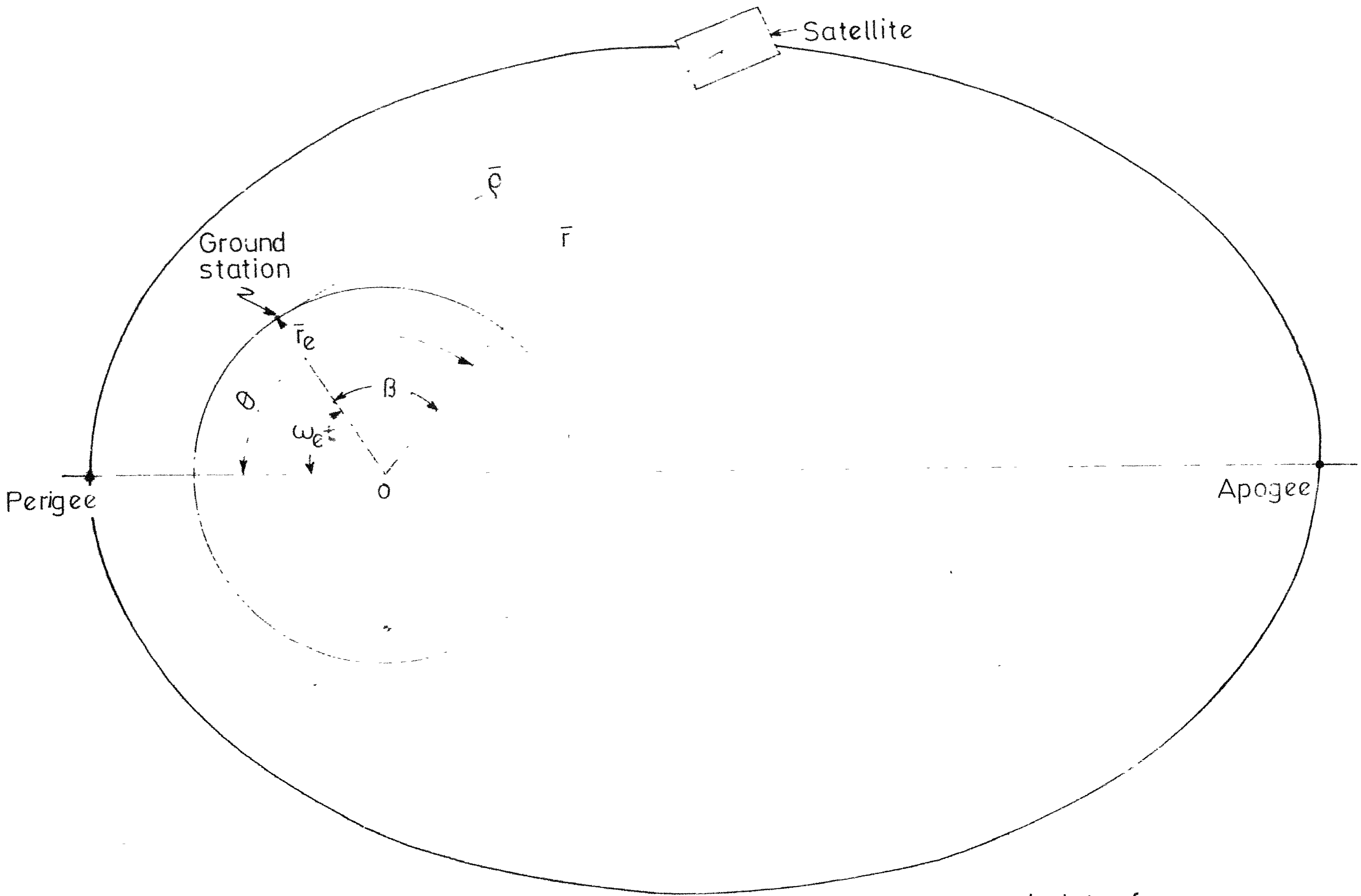


Fig.2·2 Satellite orbital geometry affecting its visibility from the ground station

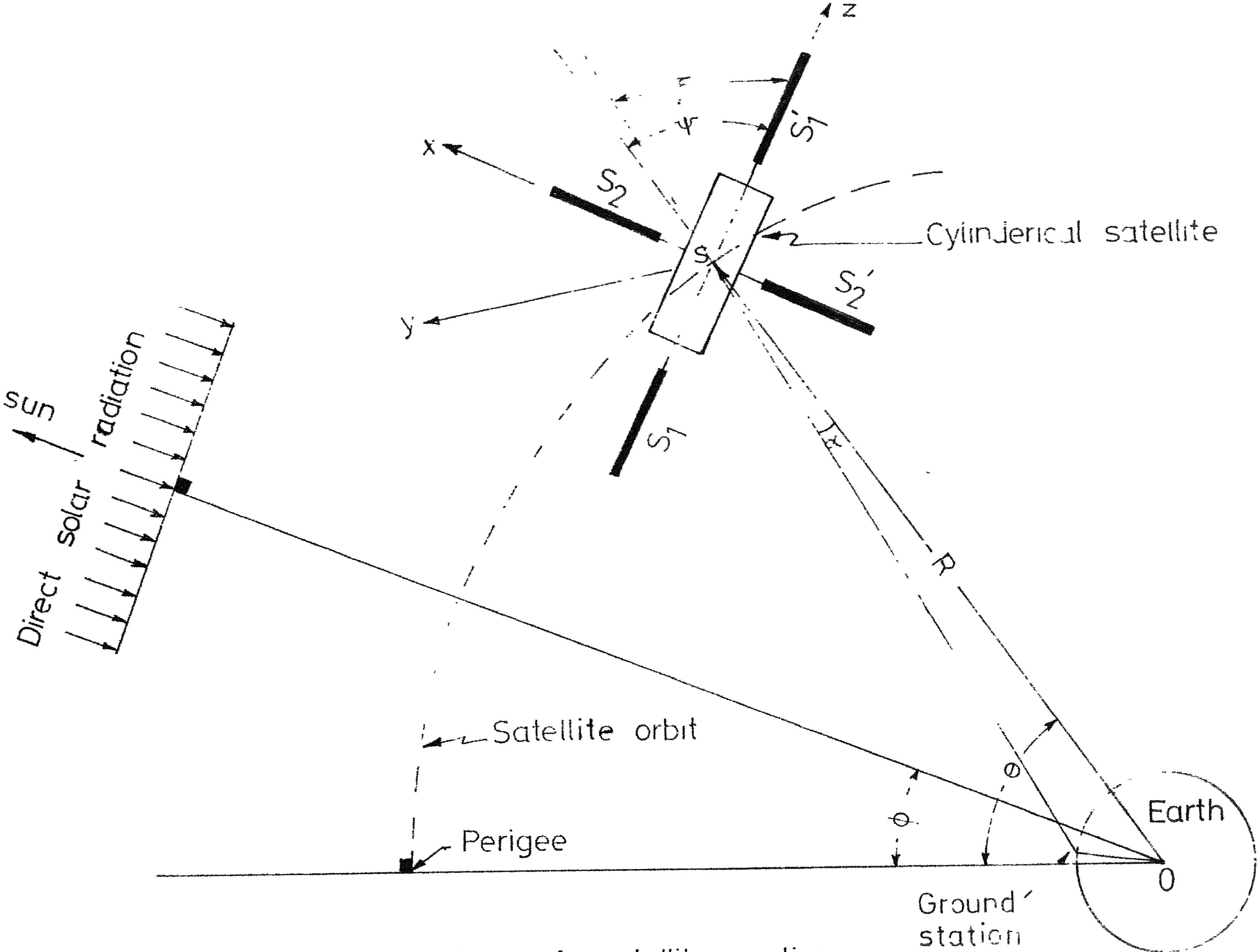


Fig.3·1 Geometry of satellite motion

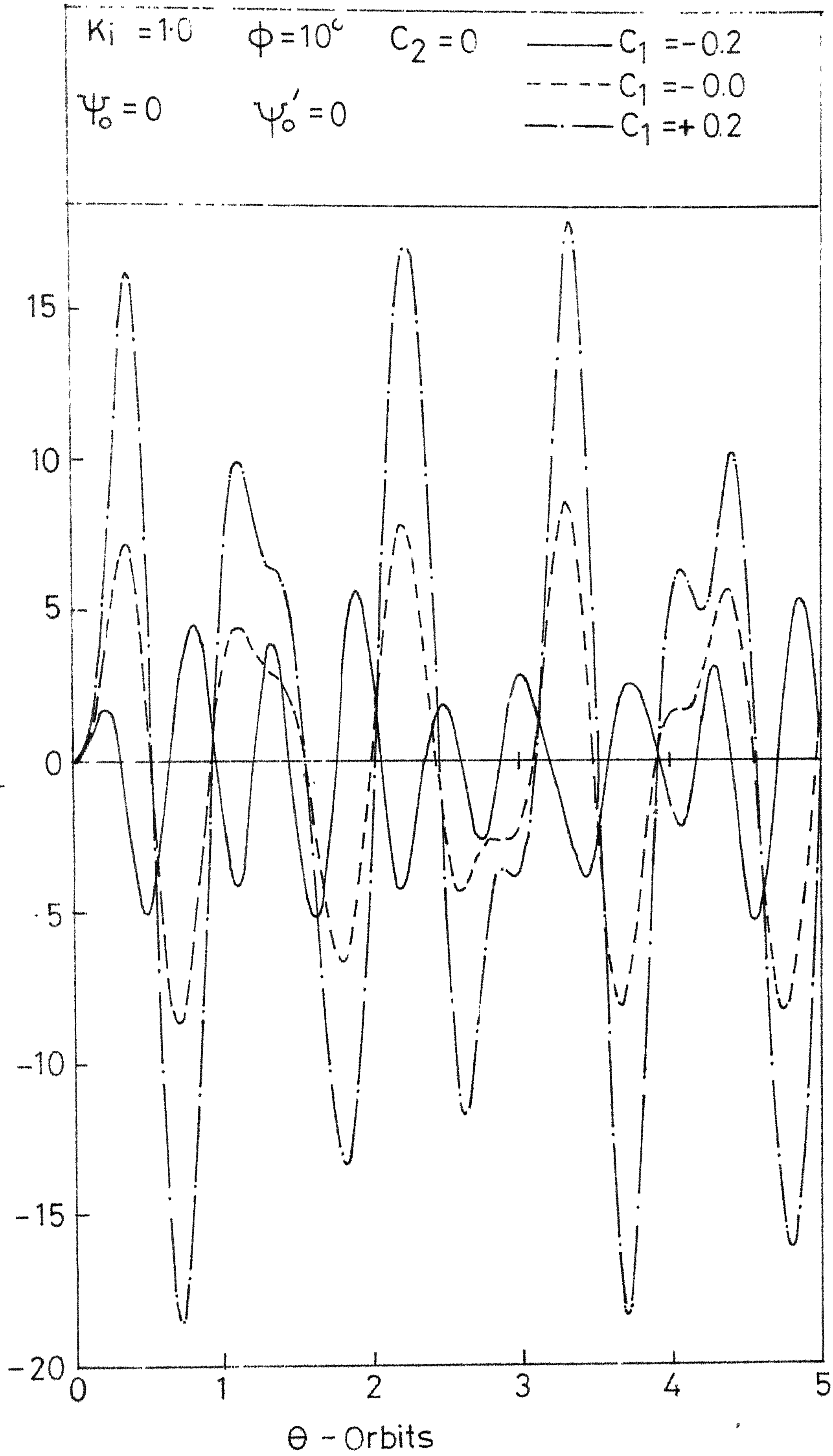


Fig. 3·2 Effect of solar radiation pressure on librational response for (a) e = 0.1

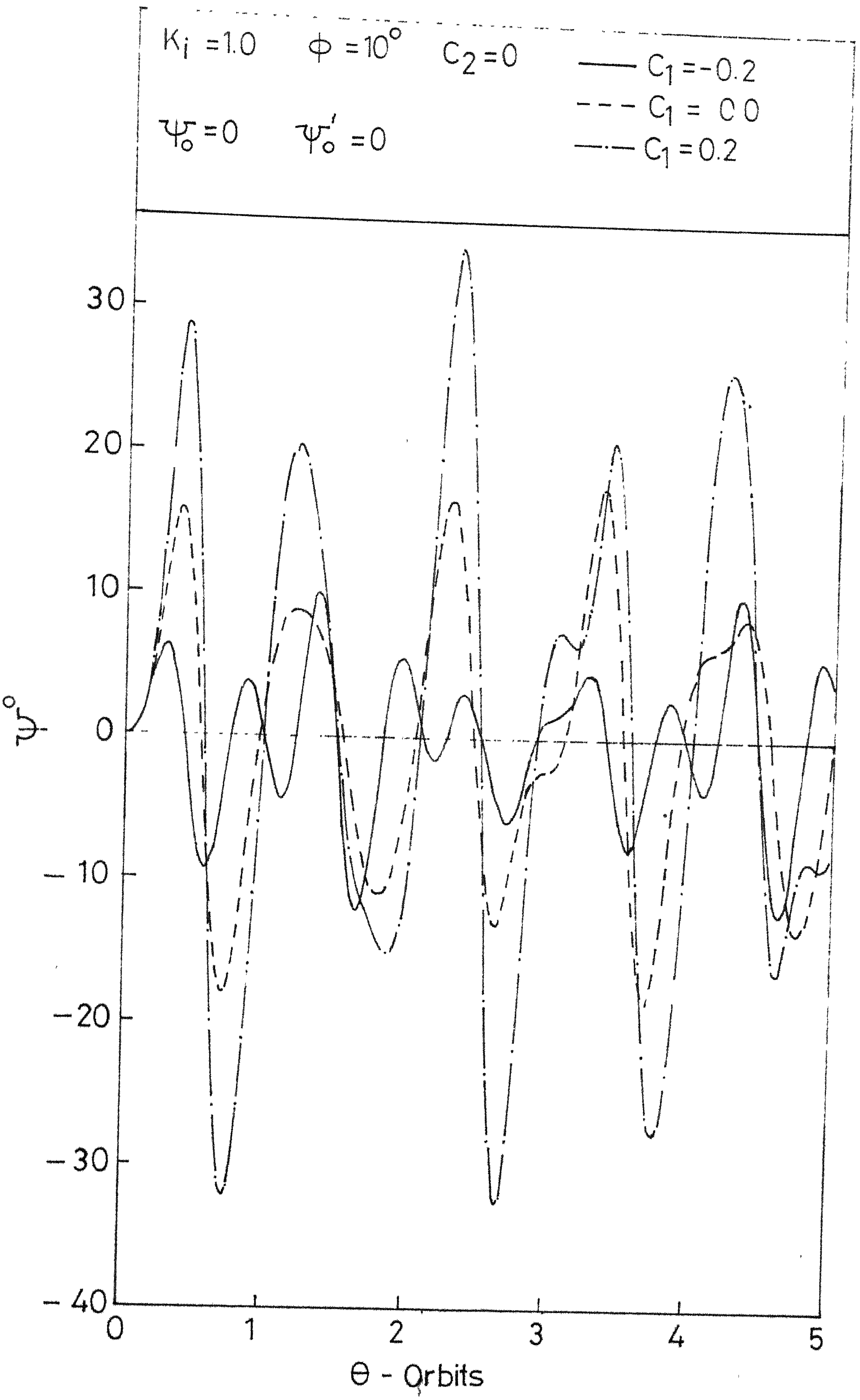


Fig.3.2 Effect of solar radiation pressure on librational response for (b) e =0.2

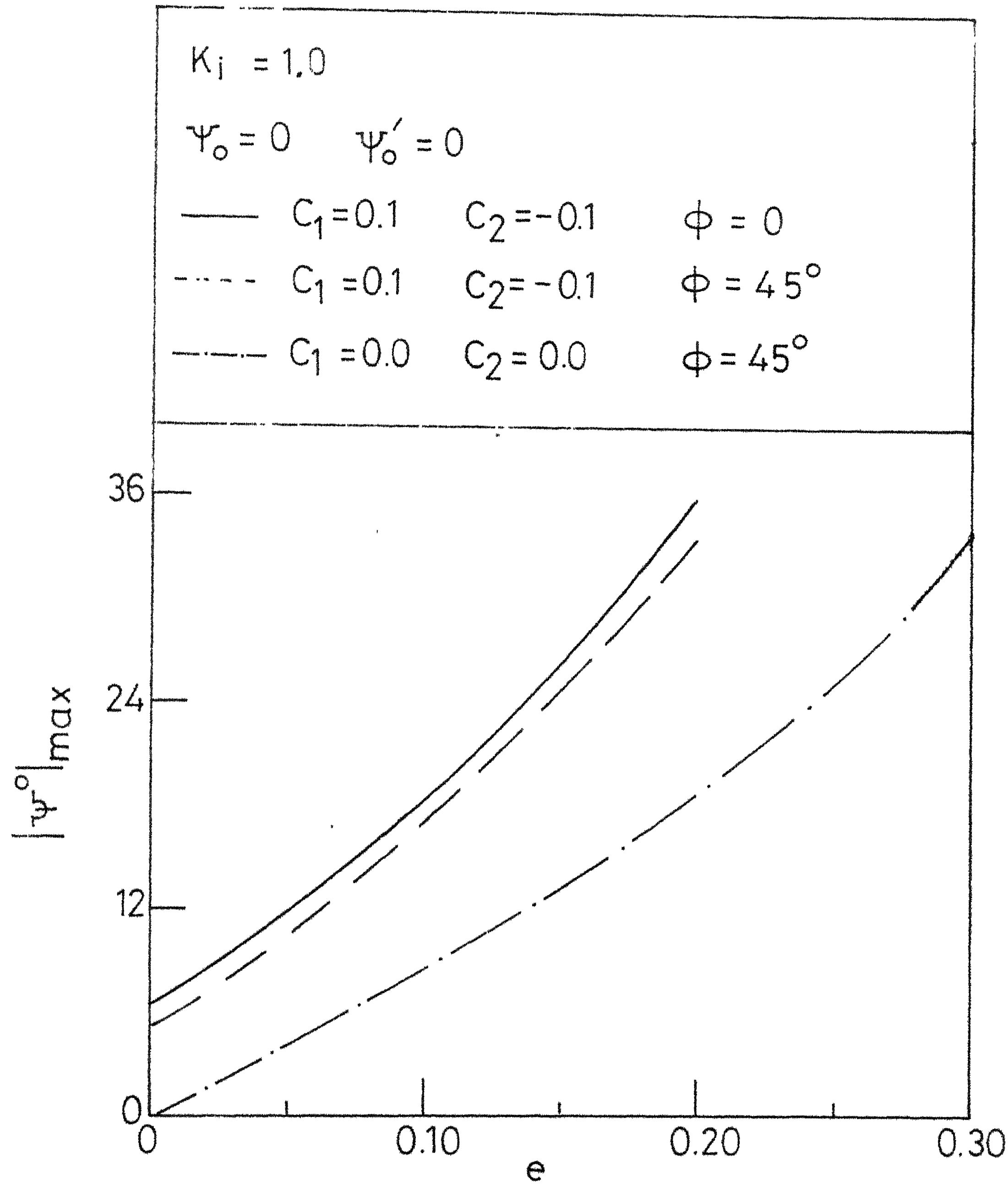


Fig. 3.3 System plots showing the effect of eccentricity on maximum librational amplitude in presence of solar radiation pressure.

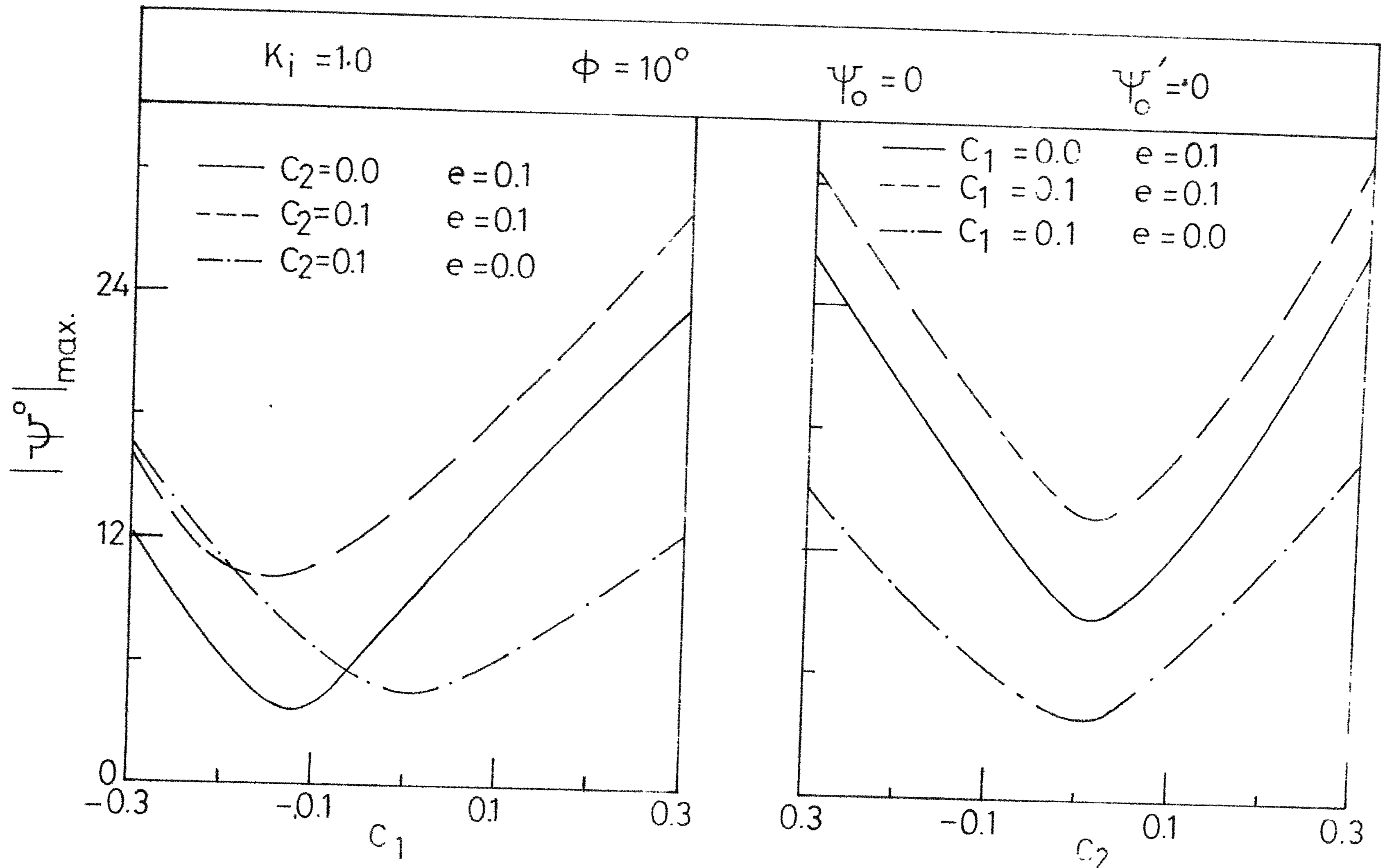


Fig.3·4 System plot showing the effect of solar parameters on maximum librational amplitude.

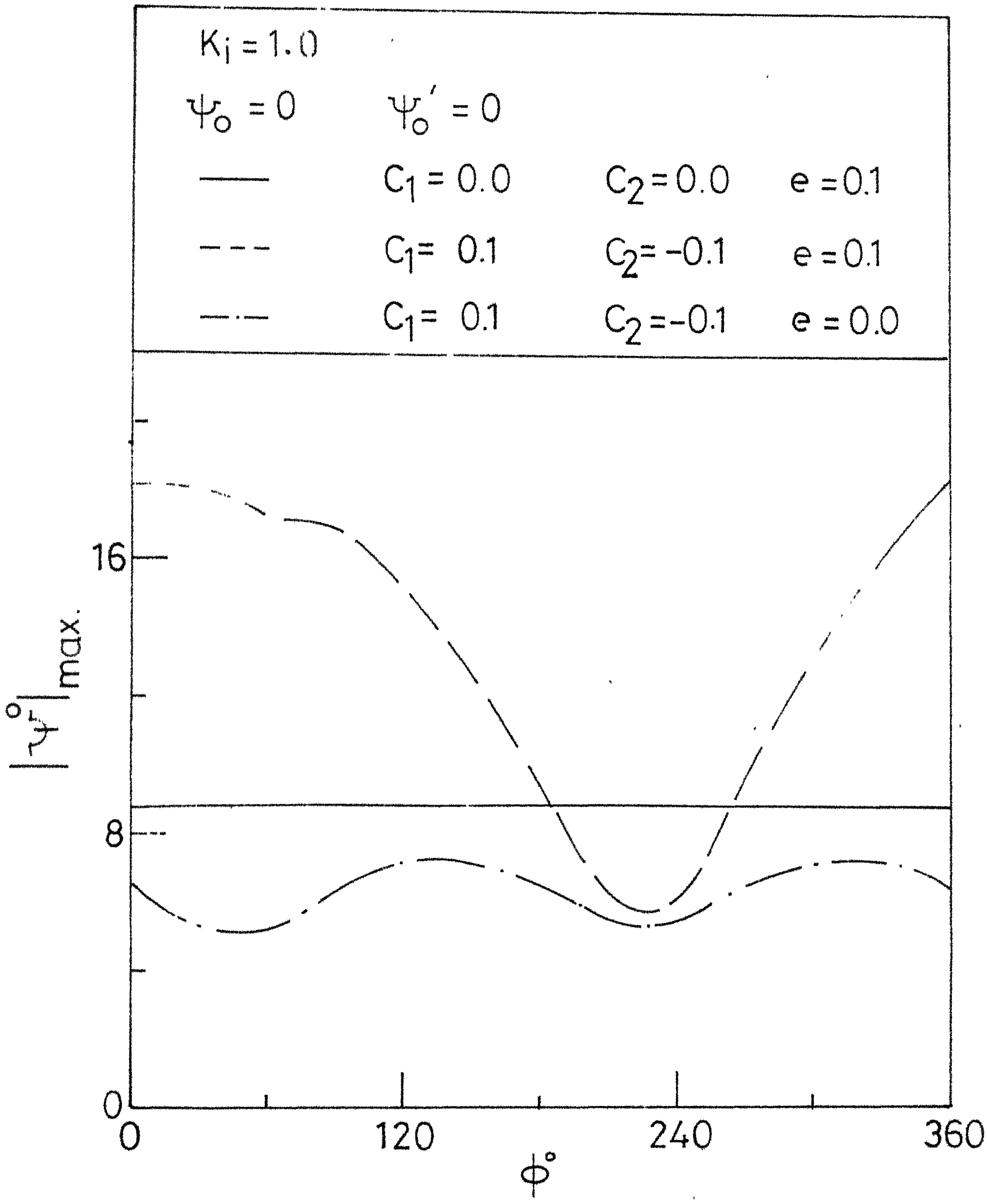


Fig.3.5 System plots showing the effect of solar position angle on maximum librational amplitude

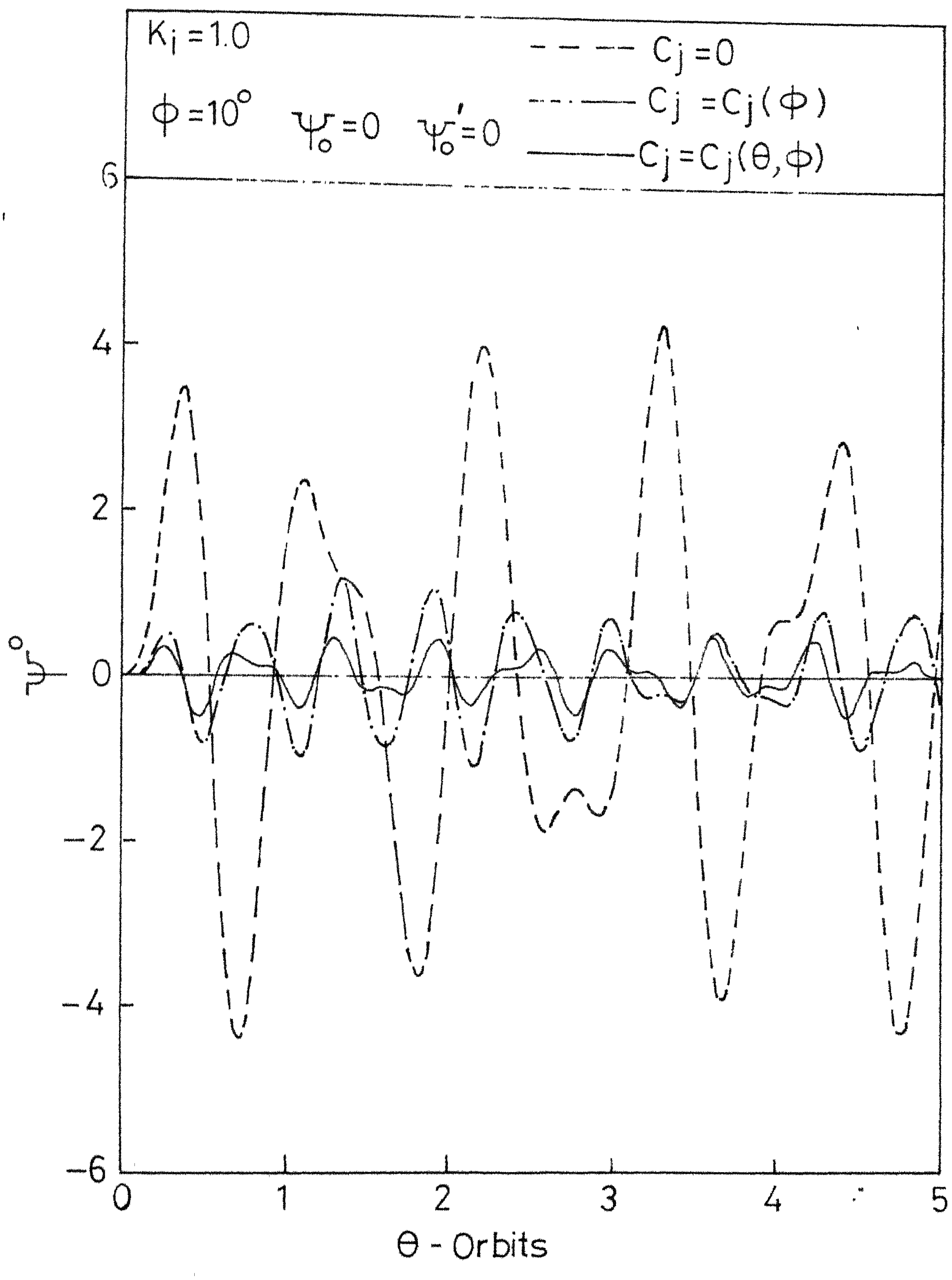


Fig.3·6 Typical satellite response showing the attitude control through the proposed solar controller for (a) e = 0.05

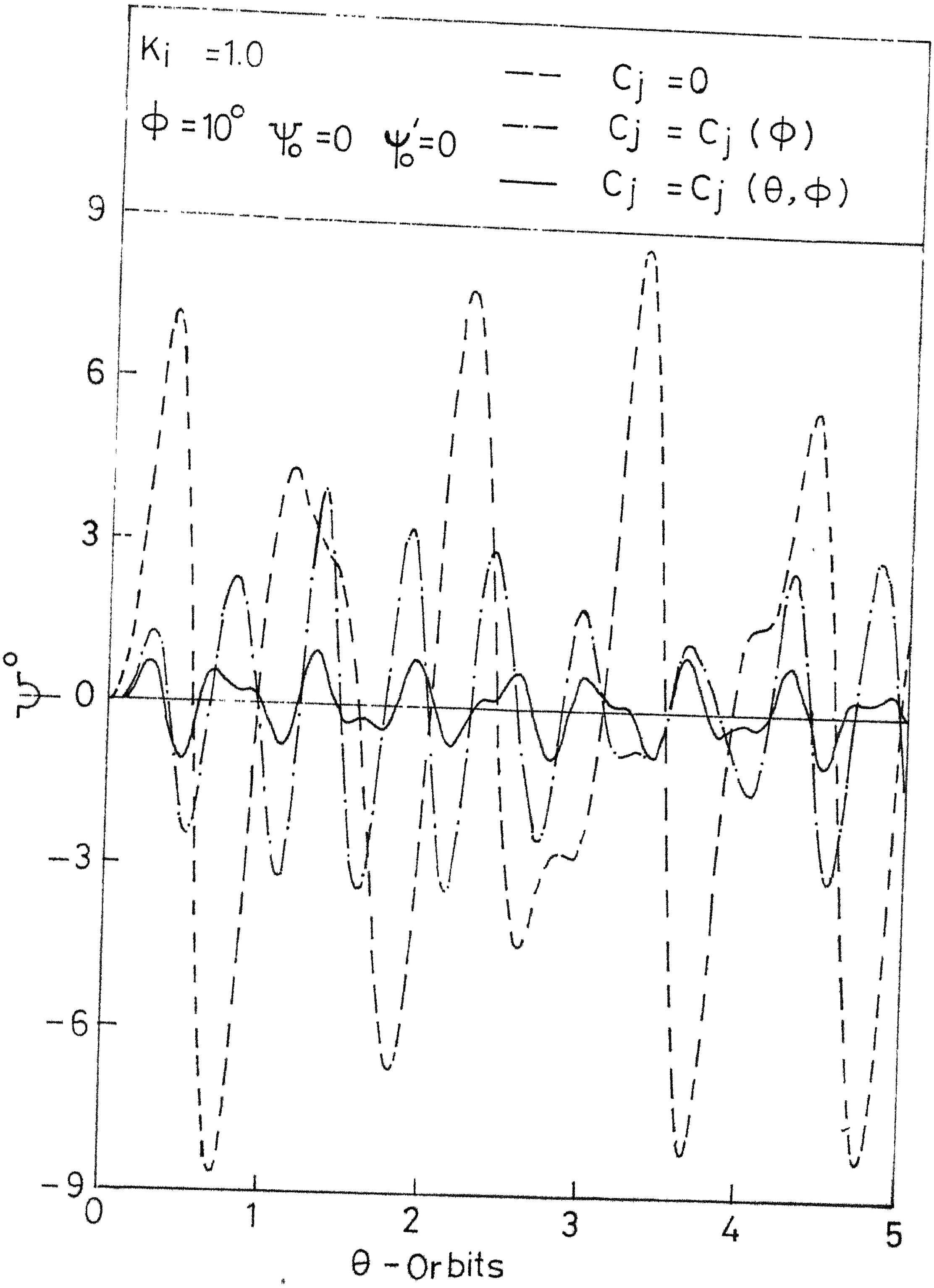


Fig. 3·6 Typical satellite response showing the attitude control through the proposed solar controller for (b) e = 0.1

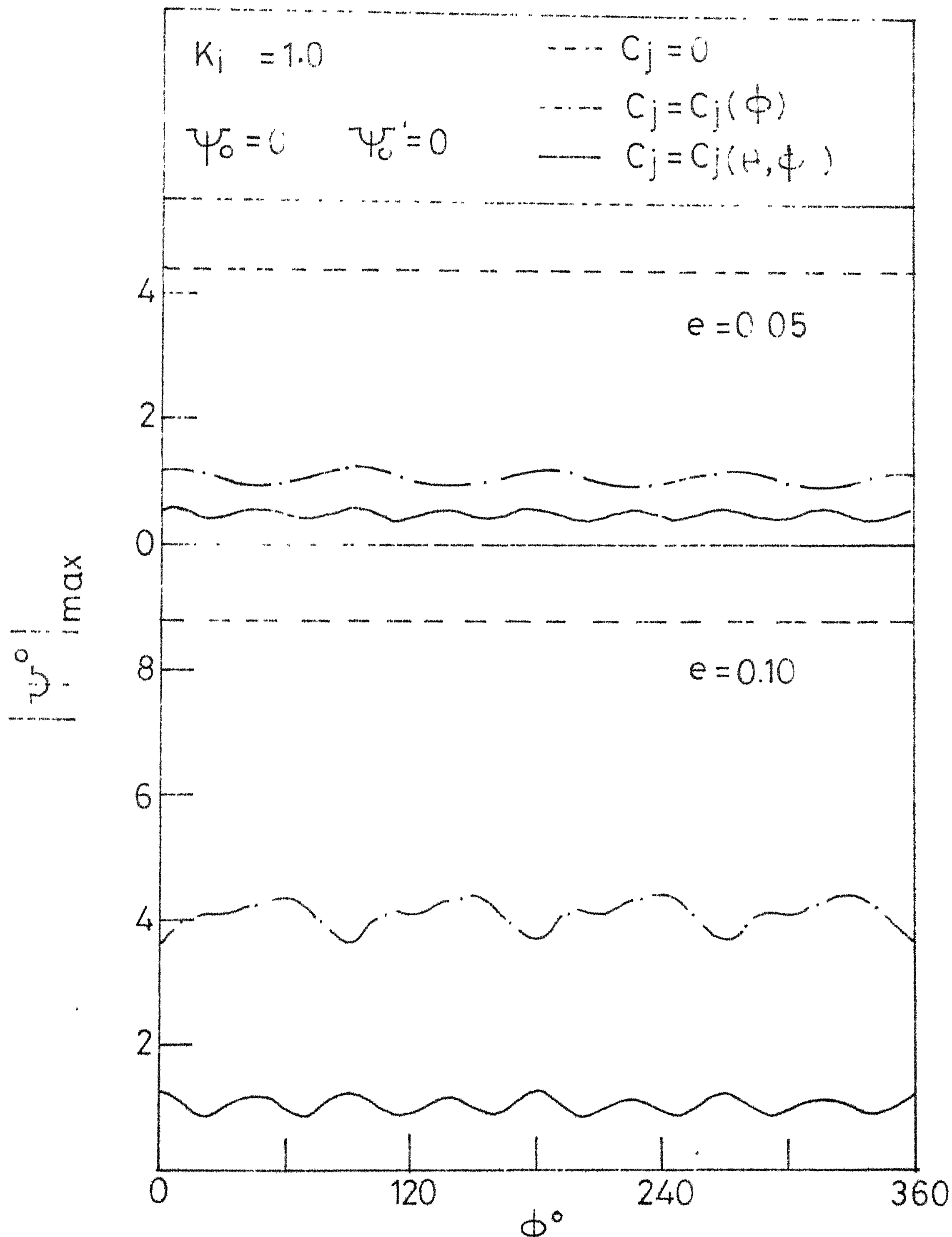


Fig.3.7 System plots showing the attitude control characterstics as affected by the solar position angle.

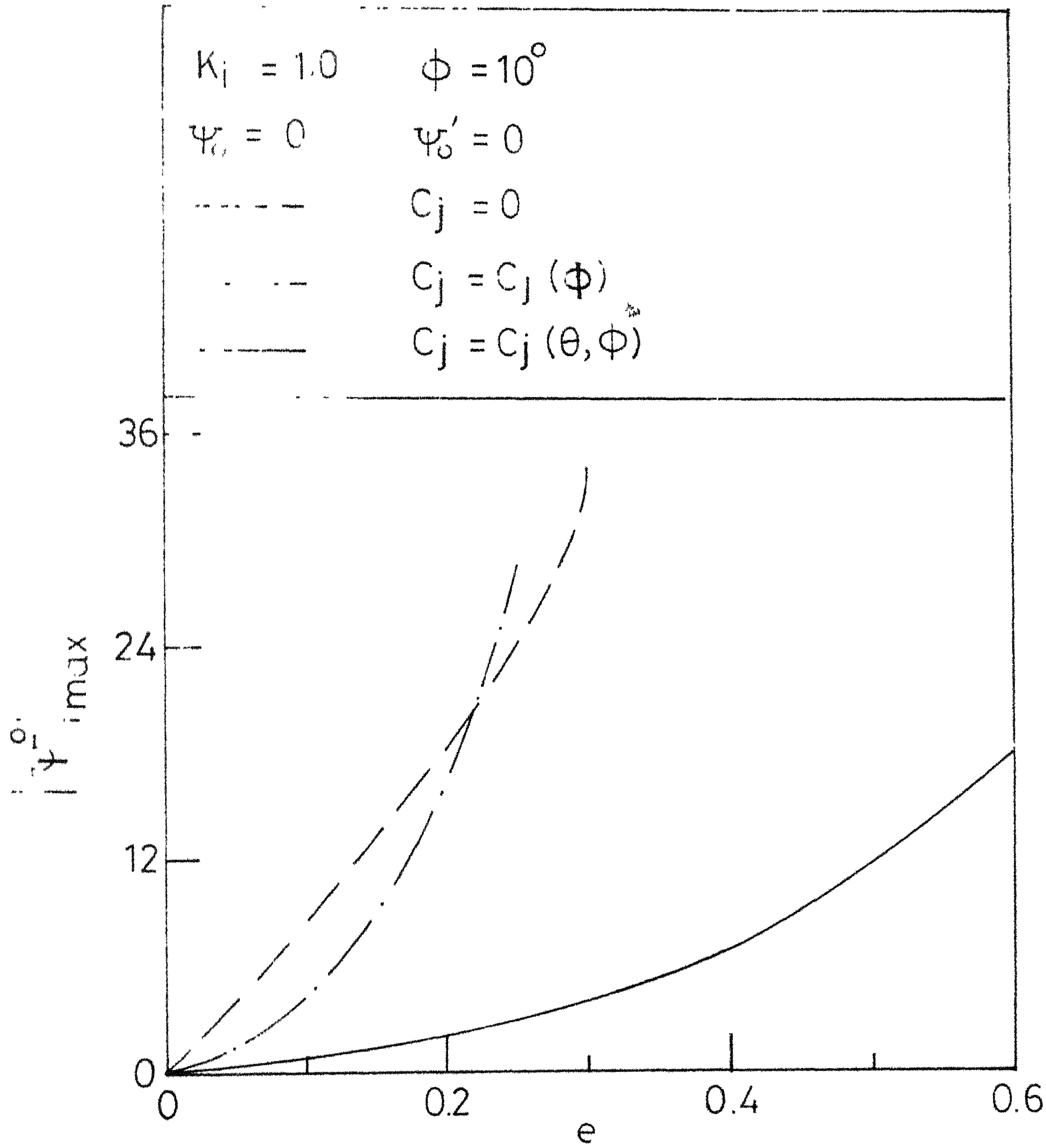


Fig.3·8 System plots showing attitude control characterstics as affected by the eccentricity.

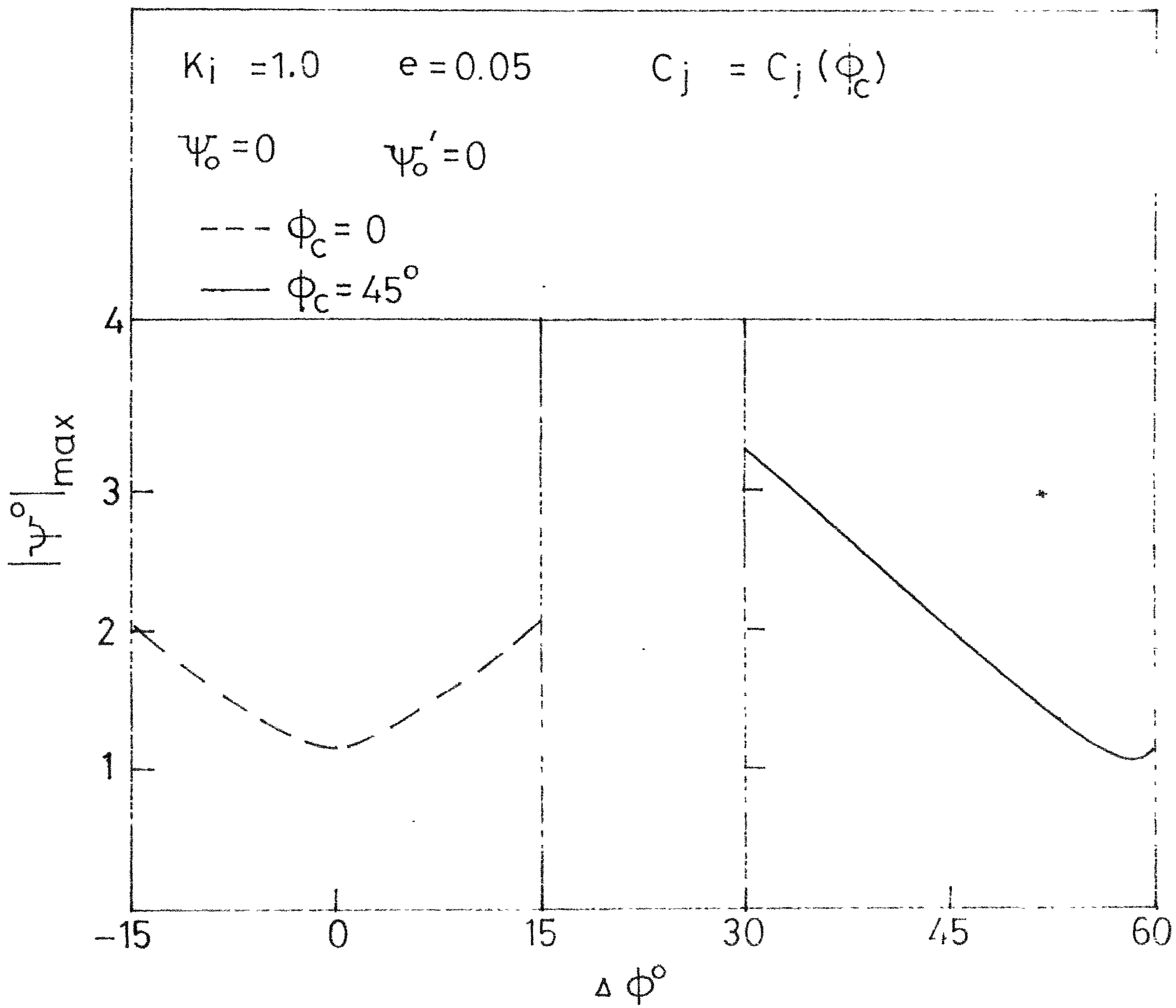


Fig.3·9 System plots showing the effect of changing the solar position angle from its nominal value

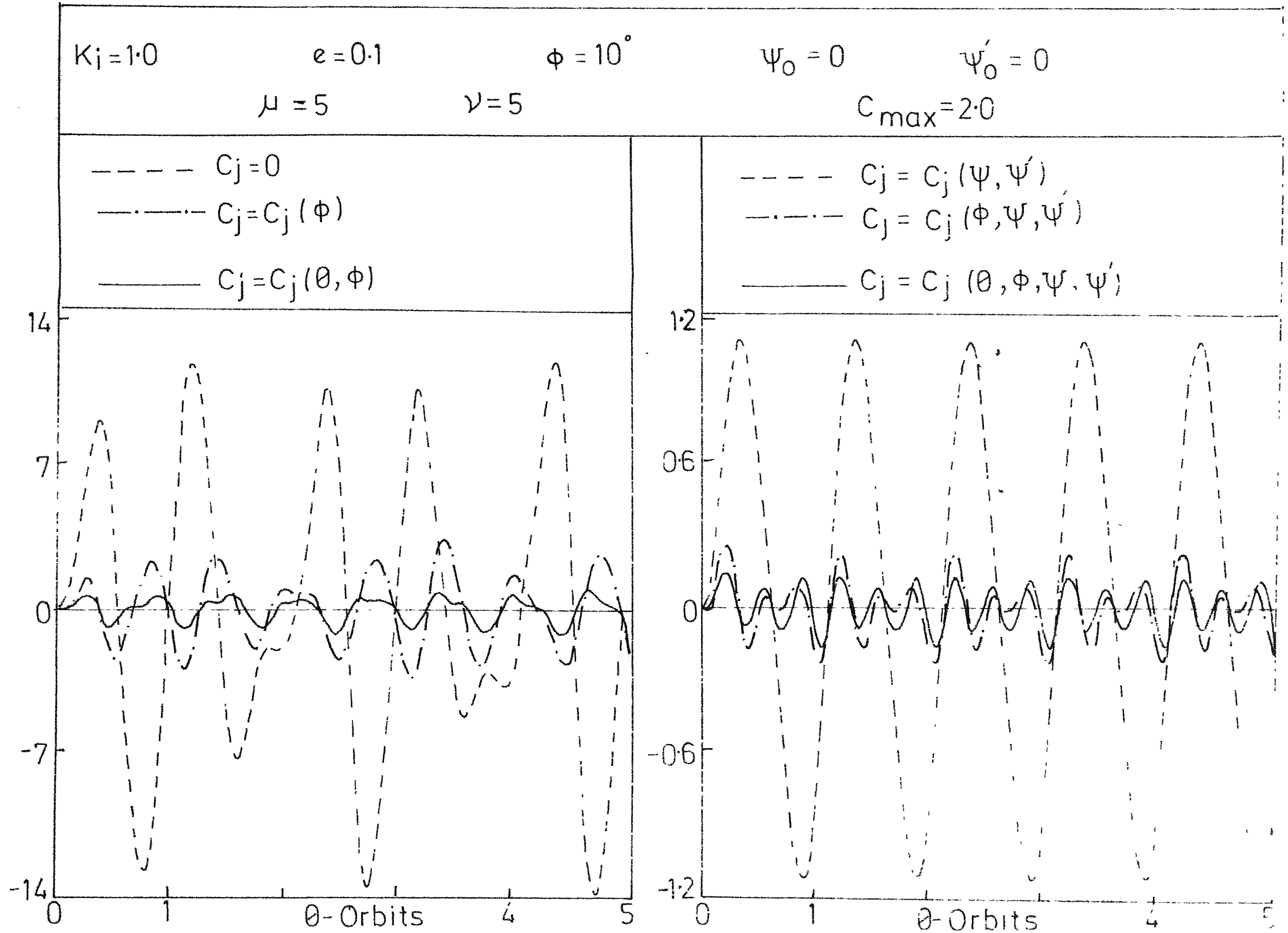


Fig. 3·10 Typical satellite response showing attitude control characteristics as affected by the various control policies

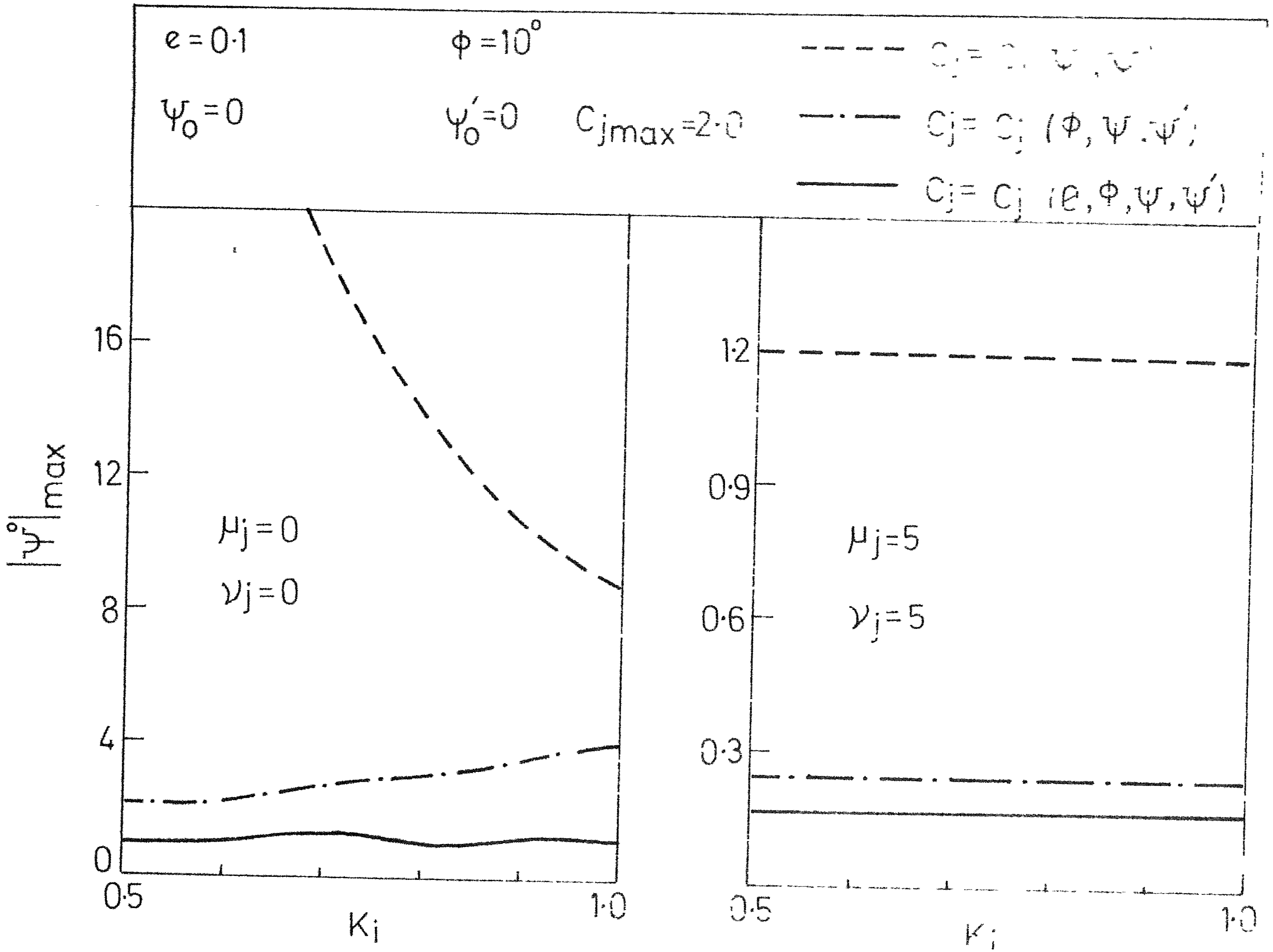


Fig. 3·11 System plots showing the effect of inertia parameter on attitude control performance

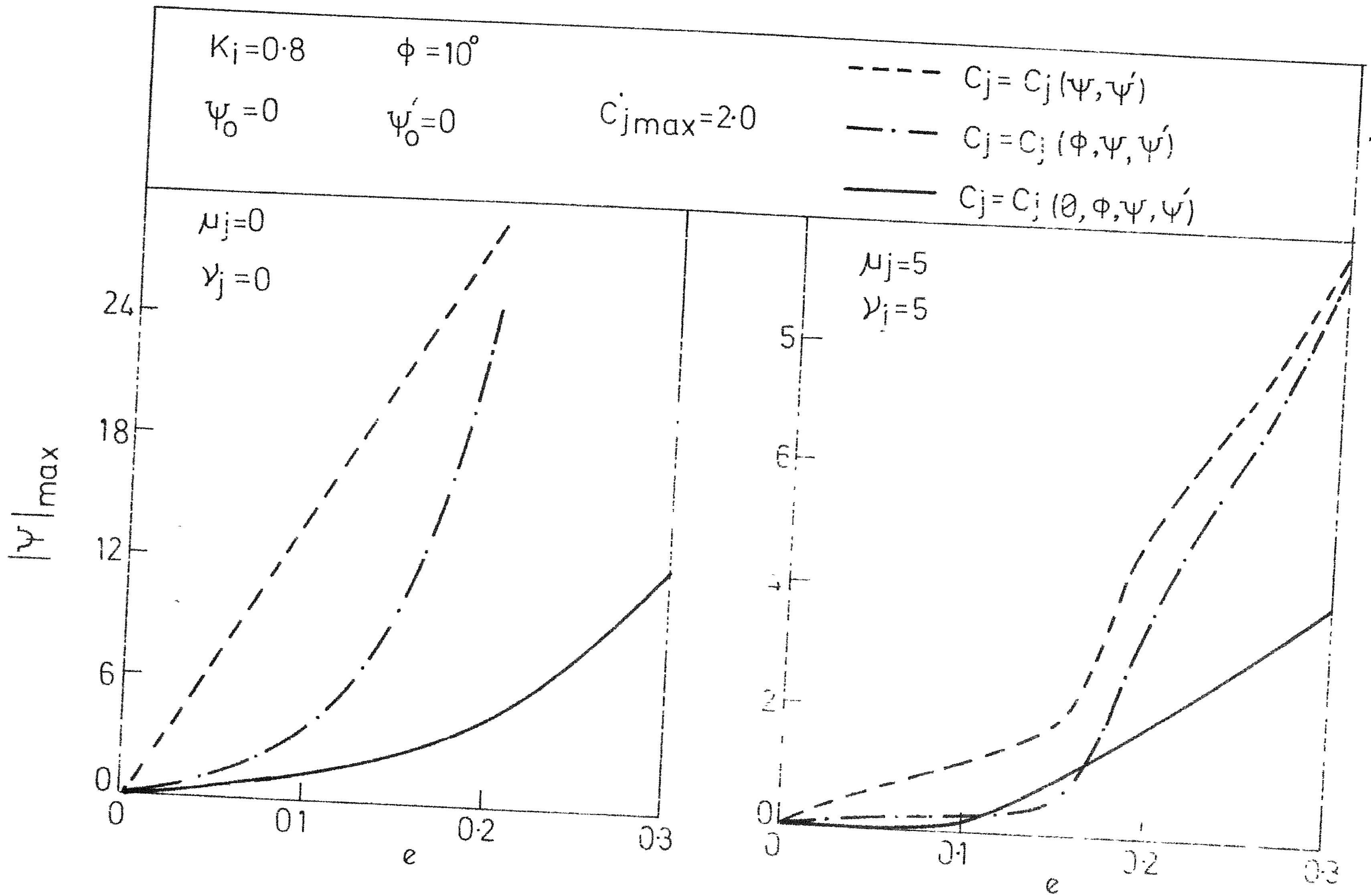


Fig.3·12 System plots showing the effect of eccentricity on attitude control performance

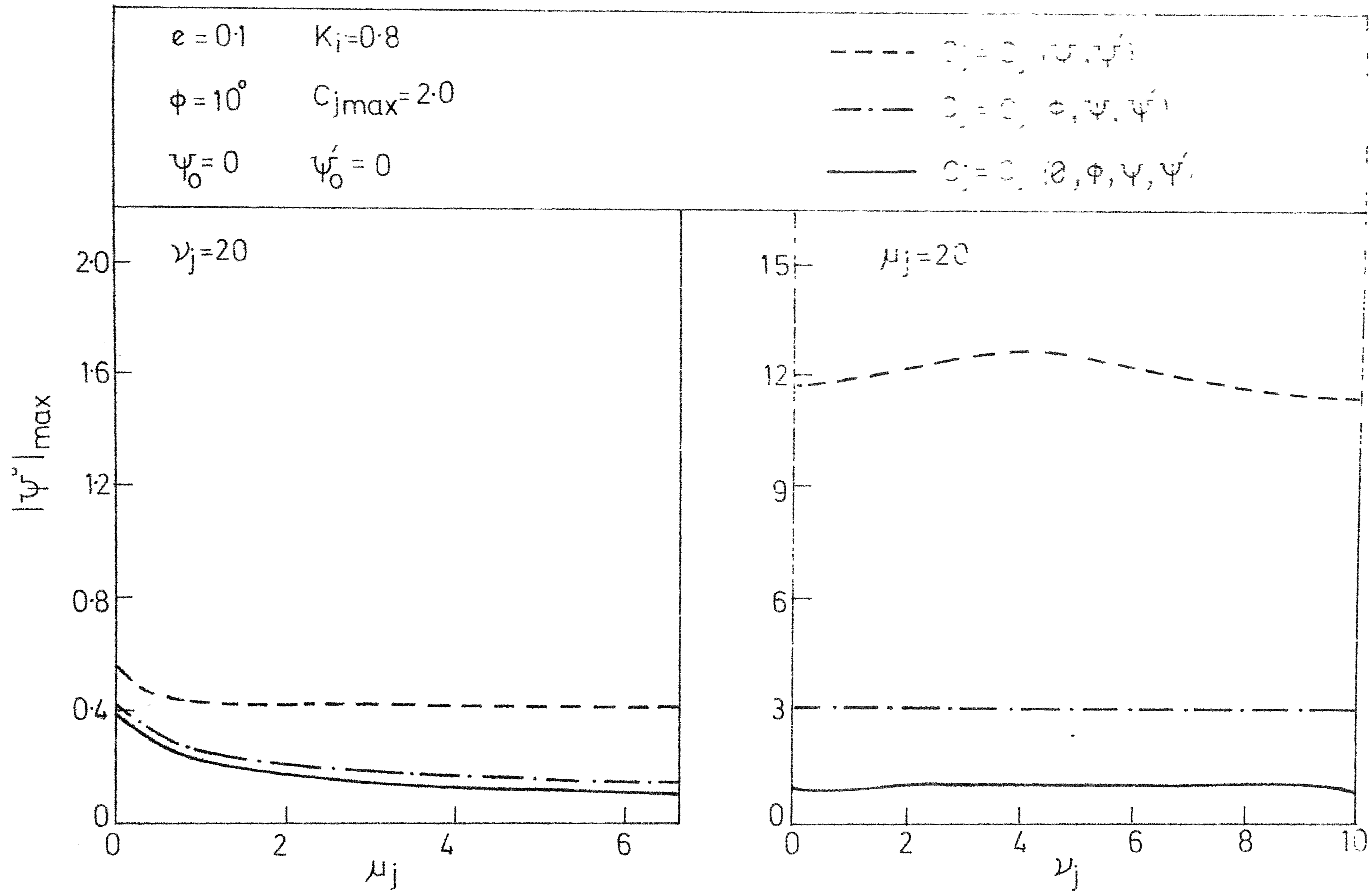


Fig.3·13 System plots showing the effect of varying control gains on attitude control performance

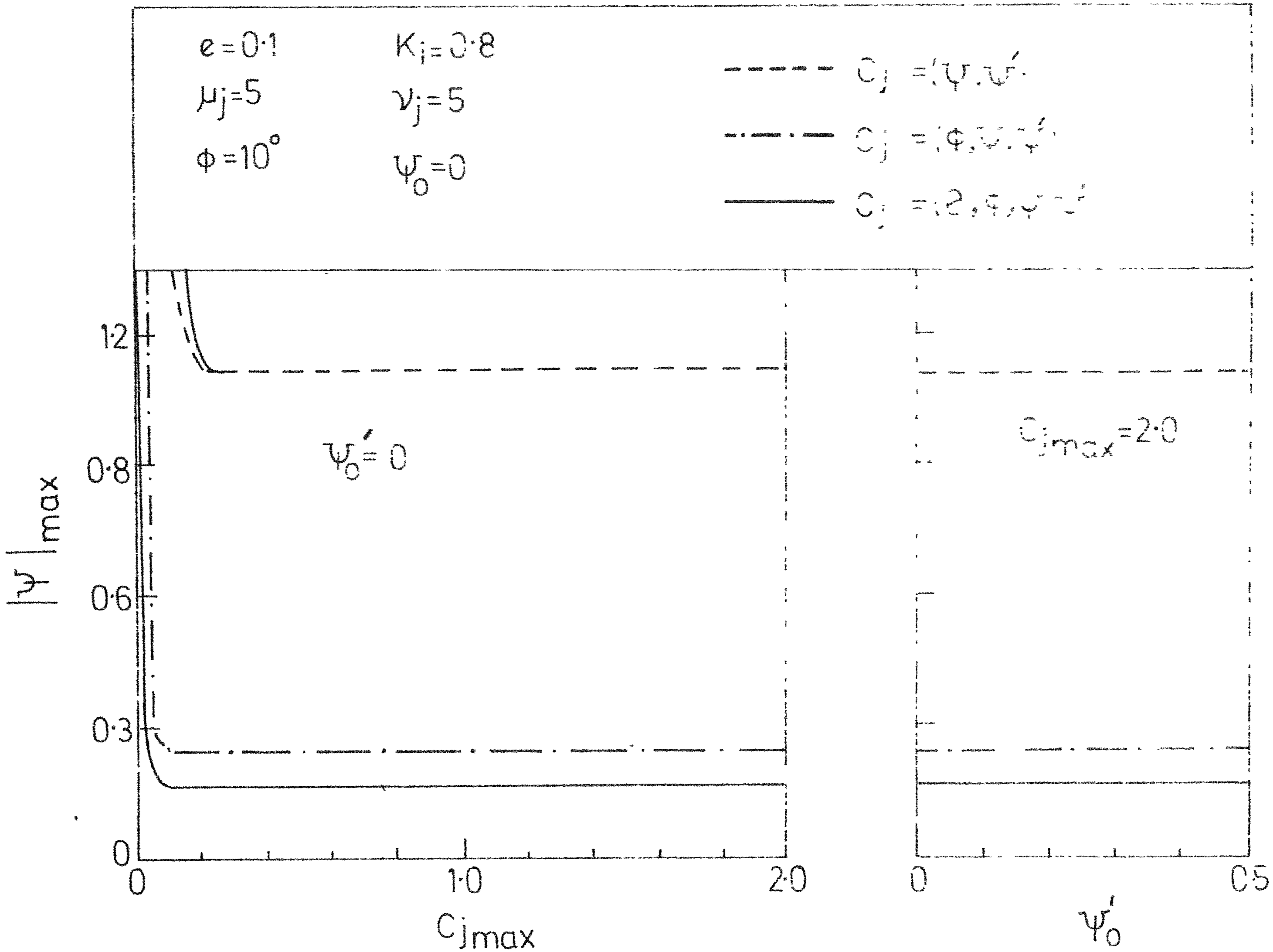


Fig. 3·14 System plots showing the effect of $C_{jmax}$ and $\psi_o'$ on attitude control performance

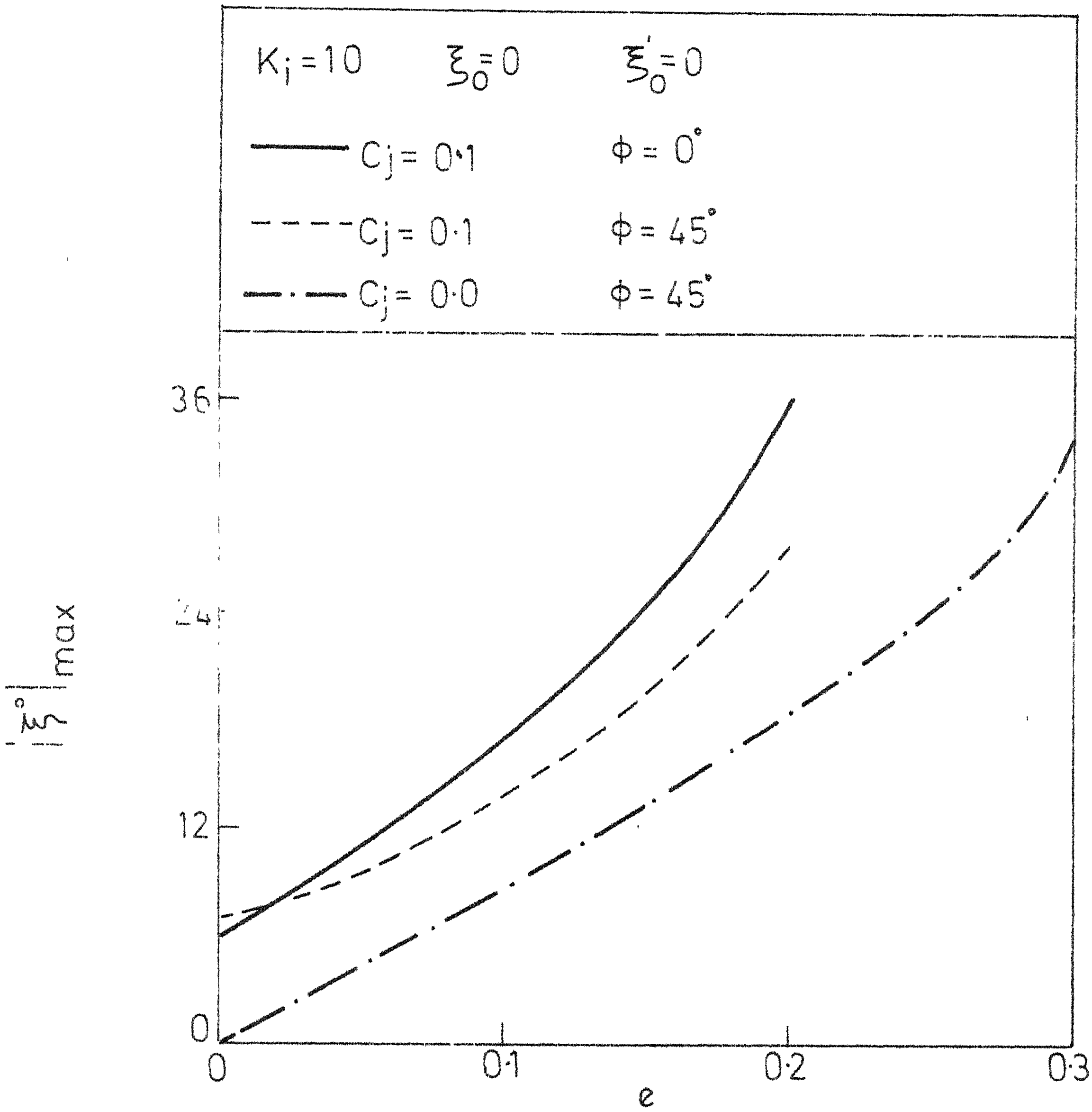


Fig. 4·1 Librational behaviour of satellite relative to the ground station as affected by eccentricity

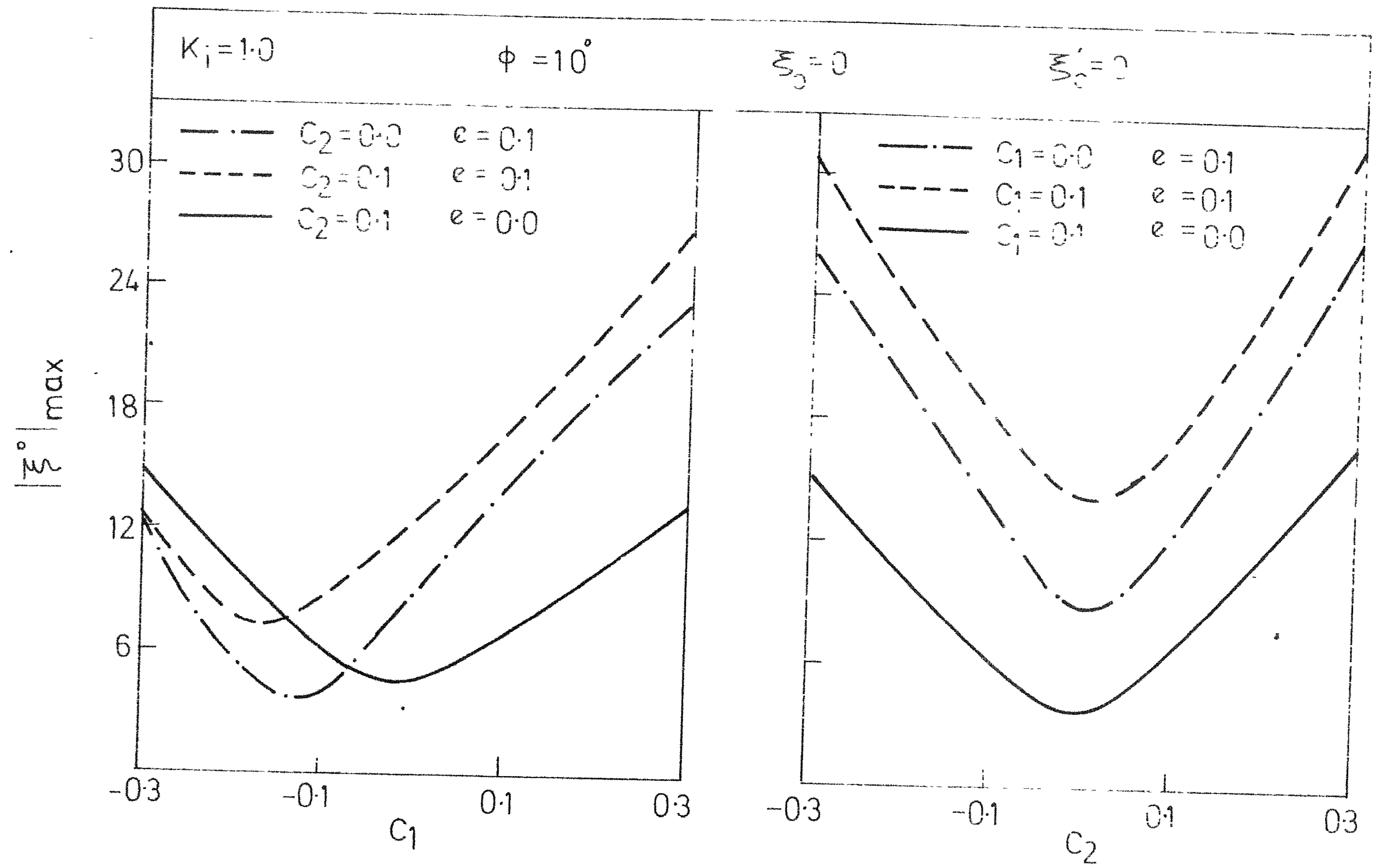


Fig.4·2 Librational behaviour of satellite relative to the ground stations as affected by solar parameters

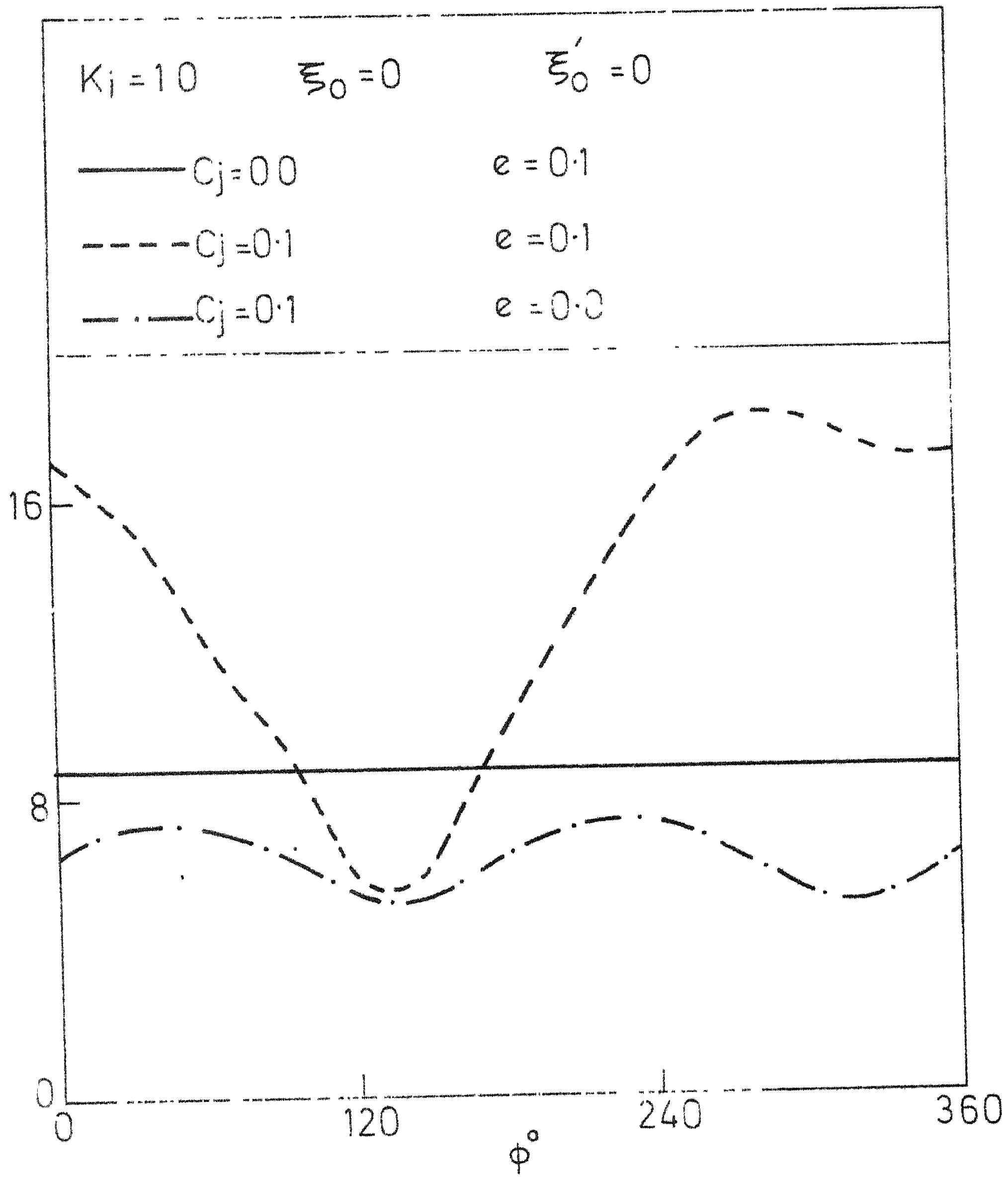


Fig 4.3 Librational behaviour of satellite relative to the ground as affected by position of the sun

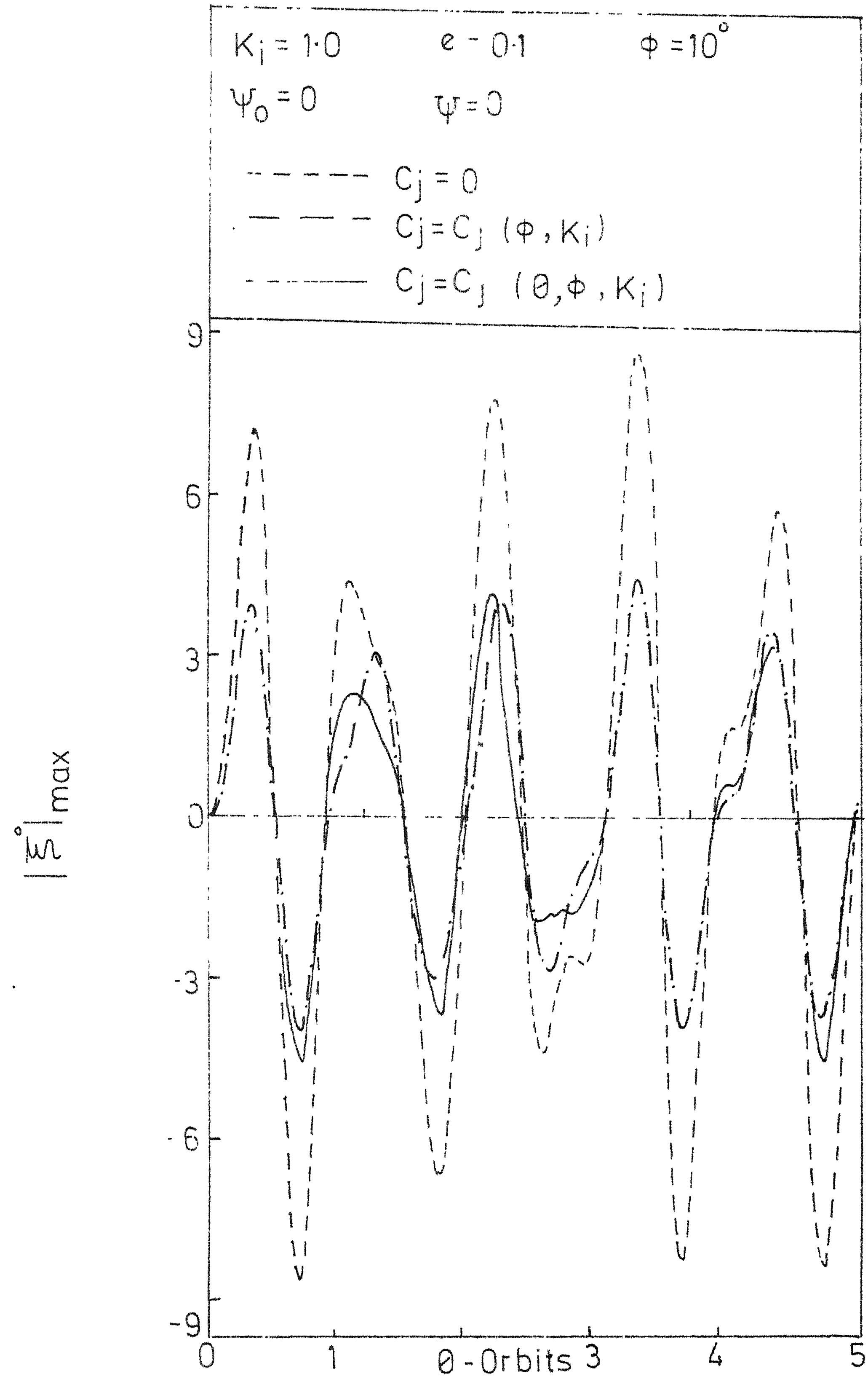


Fig.4·4 Typical satellite response showing the control of librations of satellite about line of sight as achieved through the proposed solar controller

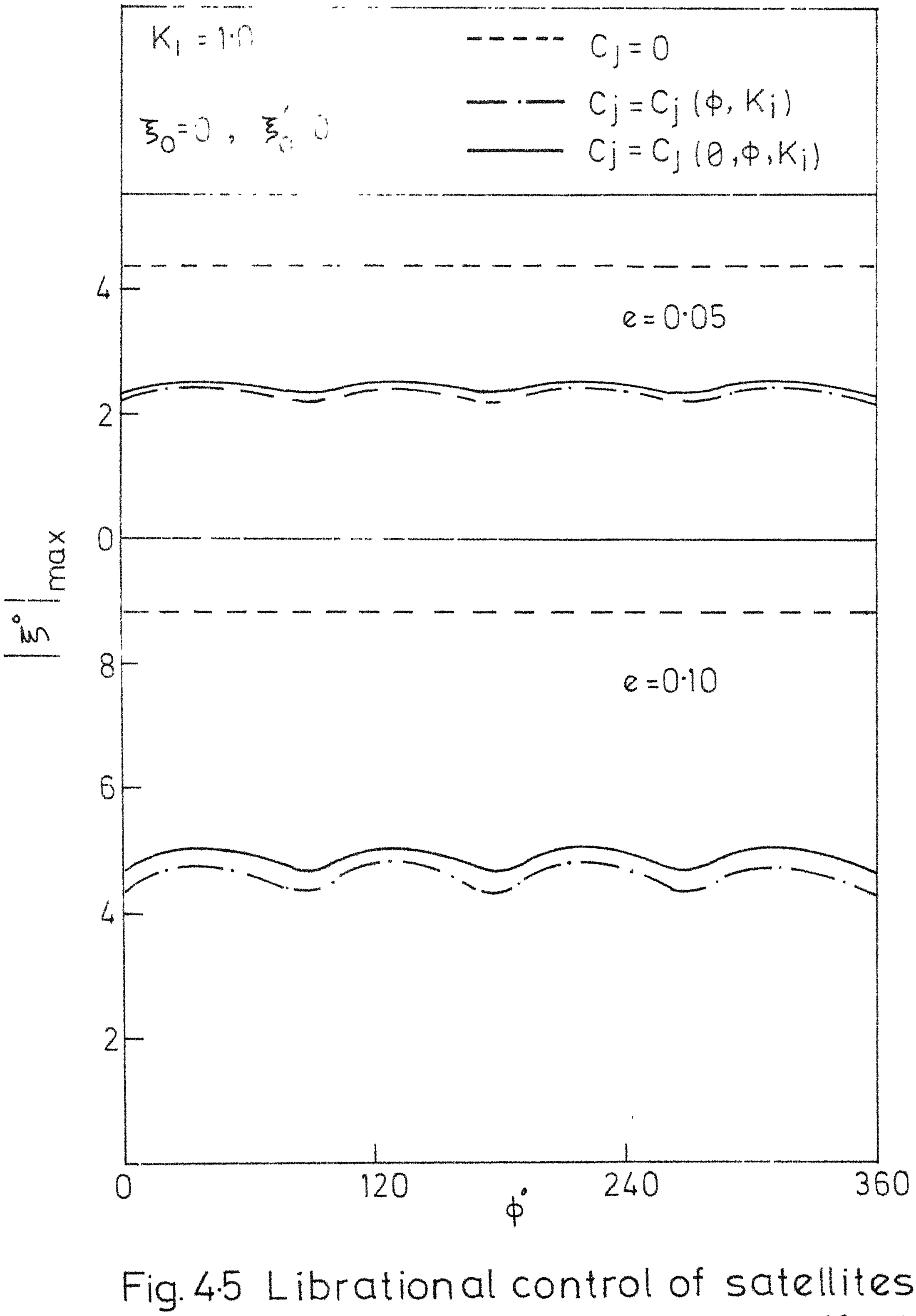


Fig. 4.5 Librational control of satellites relative to the ground station as affected by position of the sun

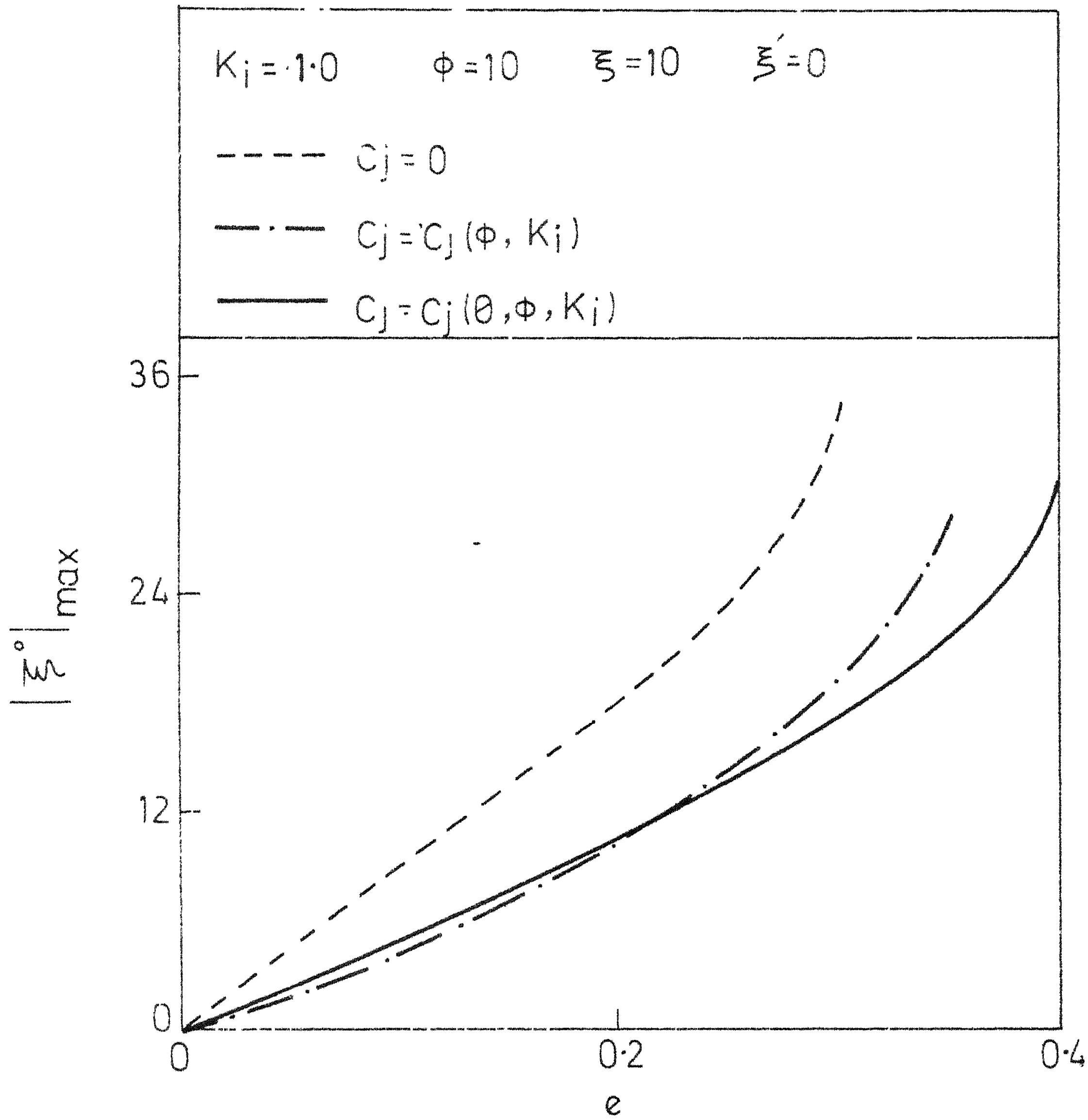


Fig.4·6 Librational control of satellite relative to the ground station as affected by the eccentricity

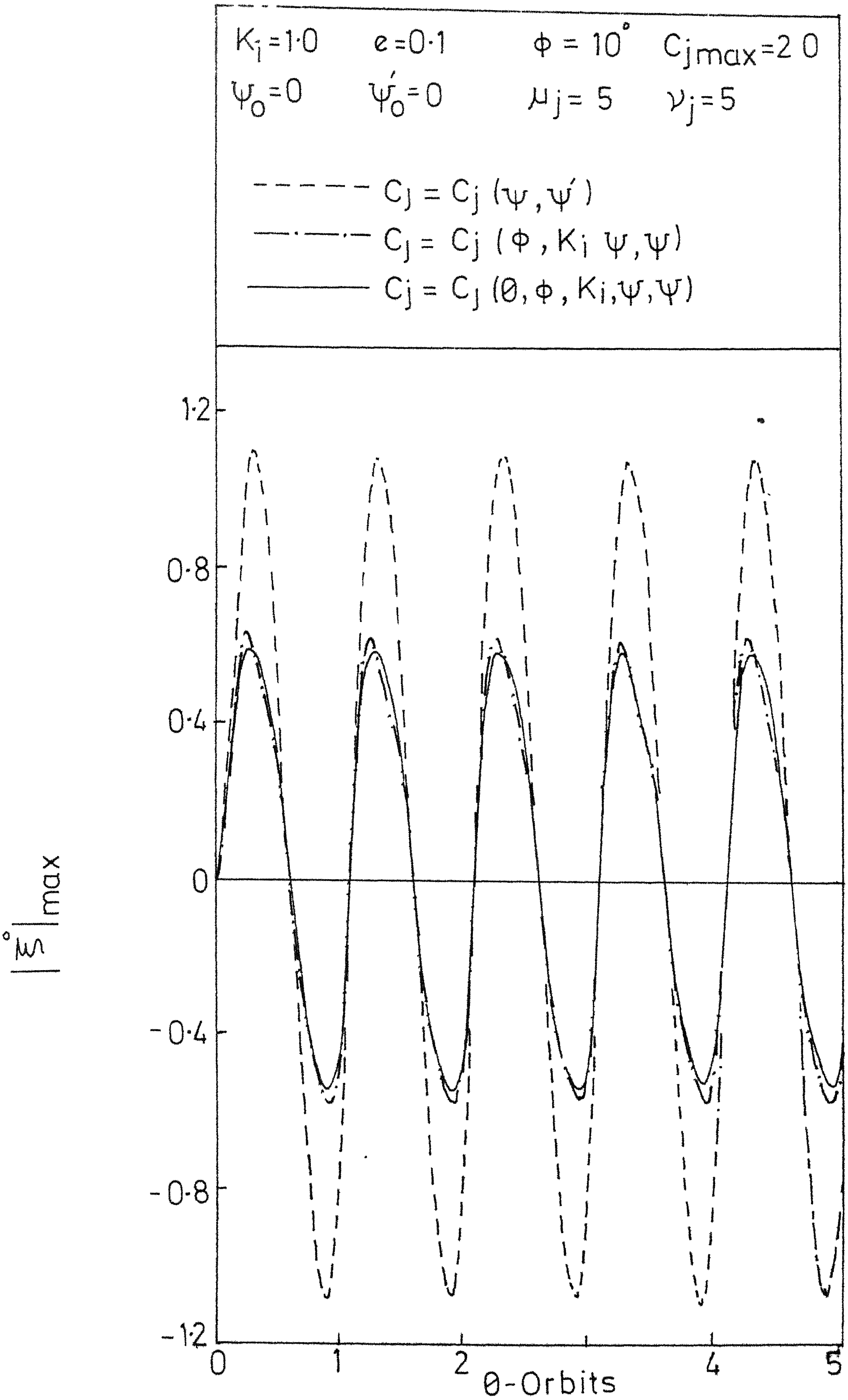


Fig. 4·7 Typical response plots showing the relative attitude control performance

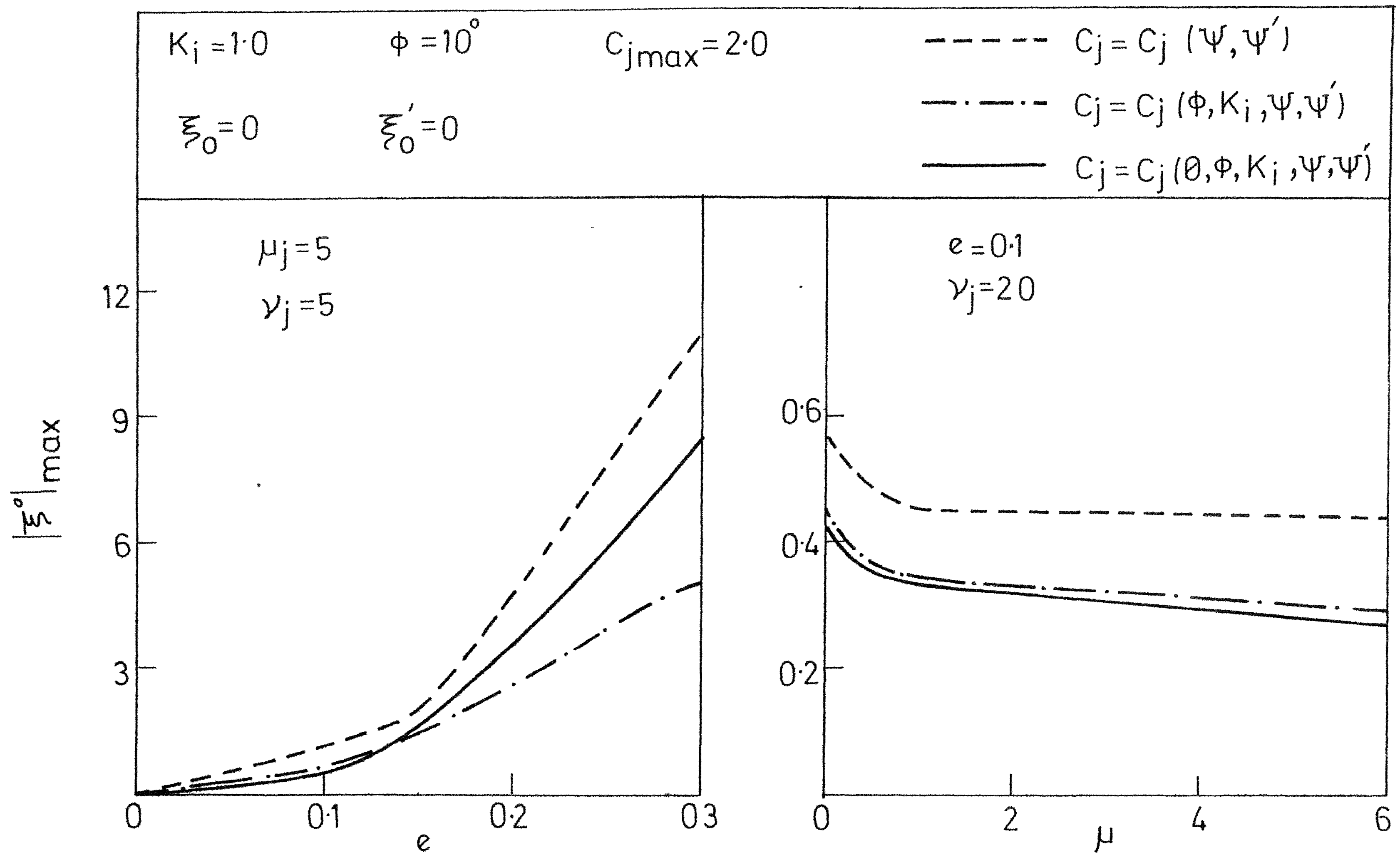


Fig. 4·8 System plots showing the effect of eccentricity and control gain $\mu$ on attitude control performance relative to the ground station

```
*IBFTC RKADE
      SUBROUTINE RKADE(X,XL,Y,Y1,L)
C  ********************************************
C  *   SUBROUTINE FOR SOLVING THE SECOND ORDER   *
C  *   ORDINARY DIFFERENTIAL EQUATION--          *
C  *        Y''=F(X,Y,Y')
C  *   X--AN ARRAY OF X-AXIS VALUES              *
C  *   XL--FINAL VALUE OF X                      *
C  *   Y--AN ARRAY OF Y-AXIS VALUES              *
C  *   Y1--DY/DX                                 *
C  *   H1--STEP LENGTH                           *
C  *   II--NO. OF LINES ALREADY PRINTED AND      *
C  *      LEFT AS BLANK                          *
C  *   JW--INPUT-OUTPUT DEVICE FOR PRINTING      *
C  *   KK=10 IF PRINTING OF VALUES IS NOT        *
C  *      REQUIRED                               *
C  *   NN--FLAG INDICATOR                        *
C  *      =0 IF X AND Y IN RADIANS               *
C  *      =1 IF X AND Y IN DEGREES               *
C  *      =2 IF X AND Y IN RADIANS AND INTEGR-   *
C  *         ATION IS FROM 0 TO 2*PI(OR MULTIPLE *
C  *         OF PI)                              *
C  *   FUNCTION F TO BE PROVIDED BY USER         *
C  *   ADAMS EXTRAPOLATION METHOD USED TO SOLVE  *
C  *   DIFFERENTIAL EQUATION                     *
C  *   RUNGE KUTTA FOURTH ORDER METHOD USED FOR  *
C  *      STARTING VALUES                        *
C  ********************************************
      REAL K1,K2,K3,K4
      DIMENSION FX(6),DEL1(6),DEL2(6),DEL3(6),DEL4(6),DEL5(6)
      DIMENSION X(L),Y(L),Y1(L)
      COMMON/VKJ/H1,II,JW,KK,NN
      PI=4.*ATAN(1.)
      PP=PI/180.
      PI2=2.*PI
      H=H1
      IF(NN.EQ.1) H=PP*H1
      H2=H*H/2.
      H3=H/2.
      N=5
      M=N+1
C ***
C *** RUNGE-KUTTA METHOD ***
C ***
      IF(NN.EQ.1) Y=PP*Y
      DO 1 I=1,N
      IF(NN.EQ.1) X(I)=PP*X(I)
      FX(I)=F(X(I),Y(I),Y1(I))
      K1=H2*F(X(I),Y(I),Y1(I))
      K2=H2*F(X(I)+H3,Y(I)+H3*Y1(I)+K1/4.,Y1(I)+K1/H)
      K3=H2*F(X(I)+H3,Y(I)+H3*Y1(I)+K1/4.,Y1(I)+K2/H)
      K4=H2*F(X(I)+H,Y(I)+H*Y1(I)+K3,Y1(I)+K3/H3)
      IF(NN.EQ.1) X(I)=X(I)/PP
      K=I+1
      Y(K)=Y(I)+H*Y1(I)+(K1+K2+K3)/3.
```

```
      Y1(K)=Y1(I)+(K1+2.*K2+2.*K3+K4)/3./H
    1 X(K)=X(I)+H1
      IF(NN.EQ.1) X(K)=PP*X(K)
      FX(K)=F(X(K),Y(K),Y1(K))
      IF(NN.EQ.1) X(K)=X(K)/PP
C ***
C *** ADAMS EXTRAPOLATION METHOD ***
C ***
    2 DO 3 I=2,M
    3 DEL1(I)=FX(I)-FX(I-1)
      DO 4 I=3,M
    4 DEL2(I)=DEL1(I)-DEL1(I-1)
      DO 5 I=4,M
    5 DEL3(I)=DEL2(I)-DEL2(I-1)
      DO 6 I=5,M
    6 DEL4(I)=DEL3(I)-DEL3(I-1)
      DEL5(M)=DEL4(M)-DEL4(M-1)
      K=K+1
      Y(K)=Y(K-1)+H*Y1(K-1)+H*H*(FX(M)/2.+DEL1(M)/6.+DEL2(M)/8.+
     1DEL3(M)*19./180.+DEL4(M)*3./32.+DEL5(M)*863./10080.)
      Y1(K)=Y1(K-1)+H*(FX(M)+DEL1(M)/2.+DEL2(M)*5./12.+DEL3(M)*3./8.+
     1DEL4(M)*251./720.+DEL5(M)*95./288.)
      DO 7 I=1,N
    7 FX(I)=FX(I+1)
      X(K)=X(K-1)+H1
      IF(NN.EQ.1) GO TO 8
      IF(NN.EQ.2) GO TO 9
      FX(M)=F(X(K),Y(K),Y1(K))
      IF(X(K)-XL) 2,2,10
    8 XX=X(K)
      X(K)=X(K)-FLOAT(IFIX(X(K)/360.))*360.
      IF(NN.EQ.1) X(K)=PP*X(K)
      FX(M)=F(X(K),Y(K),Y1(K))
      X(K)=XX
      IF(X(K)-XL) 2,2,10
    9 XX=X(K)
      X(K)=X(K)-FLOAT(IFIX(X(K)/PI2))*PI2
      FX(M)=F(X(K),Y(K),Y1(K))
      IF(X(K)-XL) 2,2,10
   10 IF(NN.NE.1) GO TO 12
      DO 11 I=1,L
   11 Y(I)=Y(I)/PP
   12 IF(KK.EQ.10) RETURN
C   ***
C   *** PRINTING THE VALUES OF X AND Y ***
C   ***
      J1=1
      J2=6
   13 II=II+1
      IF(II.EQ.70) GO TO 16
      WRITE(JW,15) (X(I),Y(I),I=J1,J2)
      J1=J1+6
      J2=J2+6
      IF(J2-L) 12,14,14
   14 J2=J2-6+1
```

```
      WRITE(JW,15) (X(I),Y(I),I=J2,L)
   15 FORMAT(1X,6(F10.1,F10.5))
      RETURN
   16 WRITE(JW,17)
   17 FORMAT(1H1,//)
      II=0
      GO TO 13
      END

*IBFTC MAX
      SUBROUTINE MAX(Y,N,YMAX)
C   *****************************************************
C   *   SUBROUTINE USED TO FIND THE MAXIMUM             *
C   *   ABSOLUTE LIBRATIONAL AMPLITUDE                  *
C   *   Y--INPUT VECTOR                                 *
C   *   N--NO. OF ELEMENTS IN VECTOR Y                  *
C   *   YMAX--MAXIMUM ABSOLUTE OF VECTOR Y              *
C   *****************************************************
      DIMENSION Y(N)
      DO 1 I=1,N
      Z=ABS(Y(I))
    1 IF(Z.GT.YMAX) YMAX=Z
      RETURN
      END

*IBFTC FS
      FUNCTION F(THETA,SI,SI1)
C   *****************************************************
C   *   FUNCTION SUB-PROGRAM USED TO SOLVE THE          *
C   *   GIVEN DIFFERENTIAL EQUATIONS USING              *
C   *   CONTROL POLICIES FOR SATELLITE TO EXECUTE       *
C   *   ABOUT THE LOCAL VERTICAL                        *
C   *   THETA--POSITION OF SATELLITE                    *
C   *   SI--LIBRATIONAL ANGLE                           *
C   *   SI1--DERIVATIVE OF LIBRATIONAL ANGLE WITH       *
C   *          RESPECT TO ANGLE THETA                   *
C   *   E--ECCENTRICITY                                 *
C   *   KI--INERTIA PARAMETER                           *
C   *   PHI--SUN POSITION ANGLE                         *
C   *   PI--CONSTANT (3.14159265)                       *
C   *   AMU1,ANU1,AMU2,ANU2--CONTROLLER GAINS           *
C   *   C1MAX,C2MAX--MAXIMUM PERMISSIBLE VALUE OF       *
C   *                 SOLAR PARAMETERS                  *
C   *   LL--FLAG INDICATOR TO USE ONE CONTROL           *
C   *         STRTAGEY AT A ONE TIME                    *
C   *   C1,C2--SOLAR PARAMETERS                         *
C   *****************************************************
      REAL KI
      COMMON E,KI,PHI,PI,AMU1,ANU1,AMU2,ANU2,C1MAX,C2MAX,LL
      PI2=2.*PI
      A=1.+E
      B=1.+E*COS(THETA)
      C= E*SIN(THETA)
```

```
      D=THETA+SI-PHI+PI2
      D=D-FLOAT(IFIX(D/PI2))*PI2
      IF(LL) 10,20,30
   10 C1=0.
      C2=0.
      GC TO 40
   20 C1=-0.75*PI*E*COS(PHI)/A**3
      C2= 0.75*PI*E*SIN(PHI)/A**3
      GC TO 40
   30 C1=-0.75*PI*E*COS(PHI)*(B/A)**3
      C2= 0.75*PI*E*SIN(PHI)*(B/A)**3
   40 C11=-(AMU1*SI1+ANU1*SI)
      IF(C11.GT.C1MAX) C11=C1MAX
      IF(C11.LT.(-C1MAX)) C11=-C1MAX
      IF(D.GT.PI.AND.D.LE.2.*PI) C11=-C11
      C1=C1+C11
      C21= (AMU2*SI1+ANU2*SI)
      IF(C21.GT.C2MAX) C21=C2MAX
      IF(C21.LT.(-C2MAX)) C21=-C2MAX
      IF(D.GT.PI/2..AND.D.LE.3.*PI/2.) C21=-C21
      C2=C2+C21
      G=(2.*C*(1.+SI1)-3.*KI*SIN(SI)*COS(SI))/B
      H=A**3/B**4*(C1*SIN(D)*(ABS(SIN(D)))-C2*COS(D)*(ABS(COS(D))))
      SI2=G+H
      F=SI2
      RETURN
      END


*IBFTC FZ
      FUNCTION F(THETA,SI,SI1)
C   ***********************************************
C   *   FUNCTION SUB-PROGRAM USED TO SOLVE THE      *
C   *   DIFFERENTIAL EQUATIONS USING DIFFERENT      *
C   *   CONTROL POLICIES FOR SATELLITE TO EXECUTE   *
C   *   ABOUT THE LINE OF SIGHT                     *
C   *   THETA--POSITION OF SATELLITE                *
C   *   SI--LIBRATIONAL ANGLE                       *
C   *   SI1--DERIVATIVE OF LIBRATIONAL ANGLE WITH   *
C   *        RESPECT TO ANGLE THETA                 *
C   *   E--ECCENTRICITY                             *
C   *   KI--INERTIA PARAMETER                       *
C   *   PHI--SUN POSITION ANGLE                     *
C   *   PI--CONSTANT (3.14159265)                   *
C   *   AMU1,ANU1,AMU2,ANU2--CONTROLLER GAINS       *
C   *   C1MAX,C2MAX--MAXIMUM PERMISSIBLE VALUE OF   *
C   *                SOLAR PARAMETERS               *
C   *   AN--CONSTANT (6.628)                        *
C   *   LL--FLAG INDICATOR TO USE ONE CONTROL       *
C   *       STRTAGEY AT A ONE TIME                  *
C   *   C1,C2--SOLAR PARAMETERS                     *
C   *   W1--THETA-WE*T                              *
C   *   WE--ANGULAR VELOCITY OF EARTH               *
C   *   T--TIME                                     *
C   ***********************************************
```

```
      REAL KI
      COMMON E,KI,PHI,PI,AMU1,ANU1,AMU2,ANU2,C1MAX,C2MAX,AN,LL
      PI2=2.*PI
      A=1.+E
      B=1.+E*COS(THETA)
      C= E*SIN(THETA)
      D=THETA+SI-PHI
      D=D-FLOAT(IFIX(D/PI2))*PI2
      G=1.-E*E
      A1=AN*G-1.
      IF(LL) 10,20,30
   10 C1=0.
      C2=0.
      GO TO 40
   20 C1=(-0.75*PI*E*COS(PHI)+2.25*PI*KI/A1*E*COS(PHI))/A**3
      C2=( 0.75*PI*E*SIN(PHI)-2.25*PI*KI/A1*E*SIN(PHI))/A**3
      GO TO 40
   30 F=1.-E
      X=(SQRT(A)-SQRT(F))*SIN(THETA)/(2.*(SQRT(A)*COS(THETA/2.)**2+
     1SQRT(F)*SIN(THETA/2.)**2))
      Y=ARSIN(2.*X/(1.+X**2))
      W1=SQRT(G)/B*C+Y
      H=PHI-W1/A1
      C1=(-0.75*PI*E*COS(H)+2.25*PI*KI/A1*E*COS(H))*(B/A)**3
      C2=( 0.75*PI*E*SIN(H)-2.25*PI*KI/A1*E*SIN(H))*(B/A)**3
   40 C11=-(AMU1*SI1+ANU1*SI)
      IF(C11.GT.C1MAX) C11=C1MAX
      IF(C11.LT.(-C1MAX)) C11=-C1MAX
      IF(D.GT.PI.AND.D.LE.2.*PI) C11=-C11
      C1=C1+C11
      C21= (AMU2*SI1+ANU2*SI)
      IF(C21.GT.C2MAX) C21=C2MAX
      IF(C21.LT.(-C2MAX)) C21=-C2MAX
      IF(D.GT.PI/2..AND.D.LE.3.*PI/2.)  C21=-C21
      C2=C2+C21
      G=(2.*C*(1.+SI1)-3.*KI*SIN(SI)*COS(SI))/B
      H=A**3/B**4*(C1*SIN(D)*(ABS(SIN(D)))-C2*COS(D)*(ABS(COS(D))))
      SI2=G+H
      F=SI2
      RETURN
      END


*IBFTC ZIVAL
      SUBROUTINE ZIVAL(THETA1,ZI,N)
C   ***********************************************************
C   *    SUBROUTINE USED TO EVALUATE THE VALUE OF            *
C   *    ANGLE ZI FROM LIBRATIONAL ANGLE SI                  *
C   *    THETA1--POSITION OF SATELLITE                       *
C   *    ZI--INPUT VECTOR SI AND RETURNS AS OUTPUT           *
C   *        VECTOR ZI                                       *
C   *    ALPHA--ANGLE BETWEEN THE LOCAL VERTICAL             *
C   *            AND THE LINE OF SIGHT                       *
C   *  N--NO. OF ELEMENTS IN VECTOR ZI                       *
C   ***********************************************************
```

```
      REAL KI
      DIMENSION THETA1(N),ZI(N)
      COMMON E,KI,PHI,PI,AMU1,ANU1,AMU2,ANU2,C1MAX,C2MAX,AN
      PI2=2.*PI
      A=1.+E
      F=1.-E
      G=1.-E*E
      DO 10 I=1,N
      THETA=THETA1(I)
      B=1.+E*COS(THETA)
      C= E*SIN(THETA)
      X=(SQRT(A)-SQRT(F))*SIN(THETA)/(2.*(SQRT(A)*COS(THETA/2.)**2+
     1SQRT(F)*SIN(THETA/2.)**2))
      Y=ARSIN(2.*X/(1.+X**2))+PI2
      W1=SQRT(G)/B*C+Y
      W1=W1-FLOAT(IFIX(W1/PI2))*PI2
      RN=AN*G/B
      ALPHA=ATAN(SIN(W1)/(RN-COS(W1)))
   10 ZI(I)=ZI(I)-ALPHA
      RETURN
      END
```